ANALYSIS OF CONSIGNMENT CONTRACTS FOR SPARE PARTS INVENTORY SYSTEMS

A THESIS
SUBMITTED TO THE DEPARTMENT OF INDUSTRIAL ENGINEERING
AND THE INSTITUTE OF ENGINEERING AND SCIENCE
OF BILKENT UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
MASTER OF SCIENCE

By
Çağrı Latifoğlu
August, 2006

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assist. Prof. Alper Şen (Advisor)

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assist. Prof. Osman Alp

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assist. Prof. Yavuz Günalay

Approved for the Institute of Engineering and Science:

Prof. Dr. Mehmet B. Baray
Director of the Institute

ABSTRACT

ANALYSIS OF CONSIGNMENT CONTRACTS FOR SPARE PARTS INVENTORY SYSTEMS

Çağrı Latifoğlu
M.S. in Industrial Engineering
Supervisor: Assist. Prof. Alper Şen
August, 2006

We study a Vendor Managed Inventory (VMI) partnership between a manufacturer and a retailer. More specifically, we consider a consignment contract, under which the manufacturer assumes the ownership of the inventory in retailer's premises until the goods are sold, the retailer pays an annual fee to the manufacturer and the manufacturer pays the retailer backorder penalties. The main motivation of this research is our experience with a capital equipment manufacturer that manages the spare parts (for its systems) inventory of its customers in their stock rooms. We consider three factors that may potentially improve the supply chain efficiency under such a partnership: i-) reduction in inventory ownership costs (per unit holding cost) ii-) reduction in replenishment lead time and iii-) joint replenishment of multiple retailer installations. We consider two cases. In the first case, there are no setup costs; the retailer (before the contract) and the manufacturer (after the contract) both manage the stock following an $(S-1, S)$ policy. In the second case, there are setup costs; the retailer manages its inventories independently following an (r, Q) policy before the contract, and the manufacturer manages inventories of multiple retailer installations jointly following a $(Q, \mathbf{S})$ policy. Through an extensive numerical study, we investigate the impact of the physical improvements above and the backorder penalties charged by the retailer on the total cost and the efficiency of the supply chain.

Keywords: Inventory Models, Vendor Managed Inventory, Joint Replenishment Problem, Supply Chain Contracts, Consignment Contracts.

ÖZET

YEDEK PARÇA ENVANTER SİSTEMLERİNDE KONŞİMENTO KONTRATLARI

Çağrı Latifoğlu
Endüstri Mühendisliği, Yüksek Lisans
Tez Yöneticisi: Yrd. Doç. Dr. Alper Şen
Ağustos, 2006

Bu tez çalışmasında, bir imalatçı ile perakendeci arasındaki Tedarikçi Yönetimli Envanter anlaşması incelenmiştir. Özellikle incelediğimiz konşimento anlaşmasında, perakendecinin tesislerindeki envanterin maliyet ve sorumluluğu yıllık bir ücret karşılığında imalatçıya geçmekte, imalatçı da yok satmalardan ötürü perakendecinin görebileceği zararları karşılamayı garanti etmektedir. Böyle bir ortaklıkta, tedarik zinciri performansını iyileştirebilecek üç faktör incelenmektedir: i-) envanter sahiplenme maliyetlerindeki azalma ii-) teslimat sürelerindeki azalma iii-) birden fazla perakende noktasının siparişlerinin ortak verilebilmesi. Bunun için iki durum incelenmektedir. İlk durumda, sipariş vermenin sabit maliyeti yoktur. Bu yüzden, hem anlaşma öncesinde hem de anlaşma sonrasında envanter yönetimi için $(S-1, S)$ politikası kullanılmaktadır. İkinci durumda ise sipariş vermenin sabit bir maliyeti vardır. Bu yüzden, anlaşma öncesinde, perakendeci noktalarındaki envanterler, perakendeciler tarafından birbirlerinden bağımsız olarak, (r, Q) politikasına göre, anlaşma sonrasında ise imalatçı tarafından ortak olarak $(Q, \mathbf{S})$ politikasına göre yönetilir. Kapsamlı bir sayısal analiz ile, bu iyileştirmelerin ve imalatçının perakendeciye yok satmalardan dolayı ödediği cezaların tedarik zinciri maliyetleri ve etkinliği üzerindeki etkileri incelenmektedir.

Anahtar sözcükler: Envanter Sistemleri, Tedarikçi Yönetimli Envanter, Toplu Sipariş Politikaları, Tedarik Zinciri Kontratları, Konşimento Kontratları.

Acknowledgement

I would like to express my most sincere gratitude to my advisor and mentor, Asst. Prof. Alper Şen for all the trust, patience and endurance that he showed during my graduate study. Without his guidance, understanding and contribution, I would not be able to make it to where I am now. I hope I can live up to his expectations from me.

I am also indebted to Assist. Prof. Osman Alp and Assist. Prof. Yavuz Günalay for excepting to read and review this thesis and for their invaluable suggestions.

I would like to express my deepest gratitude to Prof. Selim Aktürk and Prof. Mustafa Ç. Pınar for their wise suggestions and fatherly approach. I also would like to thank to all faculty members of our department for devoting their time, effort, understanding and friendship.

I want to thank Zümbül Bulut for always being there for me. I also want to express my gratitude to Ayşegül Altın for being a good friend.

I am grateful to my dear friends Evren Körpeoğlu, Fazıl Paç, Ahmet Camcı, Önder Bulut, Safa Erenay, Mehmet Mustafa Tanrıkulu, N. Çağdaş Büyükkaramıklı and Muzaffer Mısırcı for their understanding and sincere friendship. I also express thanks to all Kaytarıkçılar for their help and morale support.

Last but not the least, I wish to express my gratitude to my family. They are the most valuable for me.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

We study full service Vendor Managed Inventory (VMI) contracts for spare parts. These contracts are consignment agreements, between the manufacturer and its customers, where all decisions and services related to spare parts are assumed by the manufacturer in return for an annual fee that is paid by the customers. Ownership of the material is also assumed by the manufacturer until consumption takes place. We also investigate the Joint Replenishment Problem (JRP) for such a setting where we compare independent and joint replenishment of various installations of customers. Full service VMI contracts or consignment contracts have various potential benefits. Operational benefits of consignment contracts include reduction in cost of owning inventory, reduction in replenishment leadtime and the ability to jointly replenish multiple locations and items. Strategically, the manufacturer increases its market share and strengthens its relationships with customers by establishing such contracts. On the other hand customers receive high quality service for highly complex material while spending their effort and time on their own operations, instead of inventory and logistics management of spare parts.

Full service was defined by Stremersch [49] as comprehensive bundles of products and/or services that fully satisfy the needs and wants of a customer. The main driver of the full service contracts is the change in the products and the retailers. Short product life-cycles and time-to-market, forces companies to design, produce and market rapidly. Along with consignment, full service contracts provide the required flexibility and agility for such markets. Those contracts are usually structured considering the nature of the business and ordering procedures, receipt and issuance procedures, documentation requirements, data management requirements, place of delivery, time limits and service levels, financing and payments, qualifications and quality requirements. But the comprehensive nature of the contracts makes it difficult to assess and measure the performance of contracts. Because of that reason, the performance evaluation mechanism are also sophisticated. Various criteria such as depth of contract, scope of the contract, type of installations to maintain, degree of subcontracting, detail of information, supplier reputation, influence on performance, influence on total costs and influence on maintenance costs are used to evaluate and asses the performance of full-service contracts.

The main motivation of this research is due to our experience from a leading capital equipment manufacturer which has such a relationship with its customers. The manufacturer produces systems that perform most of the core operations in high technology material production. The customers of the company are electronics manufacturers which either use these high technology materials in their own products or sell them to other companies downstream. The capital equipment manufacturer owns research, development and manufacturing facilities in various locations such as United States, Europe and Far East which provide complex and expensive systems to world's leading electronic equipment companies. The manufacturer is at the topmost place in the related supply chain.

In our setting, the manufacturer provides spare parts of capital equipment to its customers. Capital equipments are very expensive and important investments. Cost of idle capacity due to equipment failures or service parts inventory shortages for customers is very high. For this reason the manufacturer set up a large spare parts network. This network consists of more than 70 locations across the globe, which includes 3 company owned continental distribution centers (in Europe, North America and Asia) and depots. Company also owns stock rooms as a part of spare parts network, in facilities of customers which has an agreement with the manufacturer. The distribution network is mainly responsible for procuring and distributing spare parts to depots, company owned stock rooms and customers. The depots are located such that they can provide 4-hour service to any unforeseen request. Continental distribution centers also serve orders from specific customers, orders that can not be satisfied by local depots and orders that are related to scheduled maintenance activities. Customer orders go through an order fulfillment system which searches for available inventory in different locations according to order sequence specific to each customer. The complexity of this network is further increased by more than 50,000 consumable and non-consumable parts and varying service level requirements of the customers.

Managing this immense supply chain requires a great coordination in transportation and inventory decisions. Full service contracts helps both manufacturer and its customers in coordination. We defined the operational and strategic benefits of full service contracts and those provide the required incentives to parties to participate in the agreement. There are two key observations about our supply setting: First, the manufacturer has a lower per unit holding cost than its customer since there is no additional profit margin on price of material that is incurred by customer. Also there are technical reasons, such as better preservation conditions provided for sensitive material. Second, order processing times are reduced significantly and this is enhanced with clarity of demand due to implementation of information sharing and online ordering technologies. For example, the stock rooms have a direct access to the manufacturer's ERP system under the consignment contracts.

In this research we focus on coordination issues of this complex supply chain with consignment contracts. Contracts may have different purposes such as sharing the risks arising from various sources of uncertainty, coordinating supply chain through eliminating inefficiencies (e.g. double marginalization), defining benefits and penalties of cooperative and non-cooperative behavior, building long-term relationships and explicitly clarifying terms of relationships. Also there may be different classification schemes for contracts such as specification of decision rights, pricing, minimum purchase commitments, quantity flexibility, buy back or returns policies, allocation rules, leadtimes and quality.

We consider a setting in which inventory is owned and all replenishment decisions are made by the manufacturer, and the customers pay an annual fee for this service. So the contract that we are considering is a consignment contract. Consignment may be defined as the process of a supplier placing goods at a customer location without being paid until the goods are used or sold. In practice, the manufacturer owns stock rooms in facilities of those customers where spare parts are kept. The key point that should be carefully handled in consignment contracts is the level of consigned inventory. A customer would prefer to hold a large amount of consigned inventory, since she does not have any financial obligation. The supplier, however, must determine the level at which it can provide goods profitably. Below we briefly review vendor managed inventory systems, supply chain contracts, consignment contracts and joint replenishment / inventory systems.

For lack of information, inventory is used as a proxy. In the absence of well timed and precise demand information, the lack of information is compensated with material stacks. The supplier will see batched orders from the buyer, which may not represent "true" end-customer demand. False demand signals and lack of information sharing lead to "Bullwhip Effect" which can ripple upwards in supply chain raising costs and creating disruptions. As demand information flows upwards in real time, production is more aligned with demand and supply chain performance is increased through decreasing inventories and increasing service levels. In order to achieve increased supply chain performance, VMI concept focuses on control of decision maker and ownership rights. The decision maker controls the timing and size of orders to provide benefits. Under VMI, the vendor has a certain level of responsibility of inventory decisions of customers with whom she has such a VMI partnership. In the simplest form, VMI is the practice that vendor assumes the task of generating purchase orders to replenish a customer's inventory. VMI partnerships may arise at any point of supply chain. For example, it can be between manufacturer and wholesale distributor, wholesale distributor and retailer, manufacturer and end-customer. In a VMI partnership there are varying degrees of collaboration. In the most primitive type, vendor and buyer share data and jointly develop forecasts and/or production schedules amongst supply chain partners. In a more advanced form of VMI partnership, activity and costs of managing inventory are transferred to supplying organization and this type of partnership is closer to our model. In the most advanced form, constraints and goals of customer and supplier are integrated under the guidance of market intelligence provided by the supplier to achieve better supply chain performance. Hausman [30] introduced the “Supplier Managed Availability” concept, which states that inventory at downstream site is not an aim itself but just an enabler of sales or production activity. There are other methods to provide “availability” other than stocking inventory such as using faster modes of transportation and producing faster. Supplier managed availability concept is similar to VMI in spirit. Under VMI, service level to end customer, sales, return on assets increases while routine replenishment activities and fulfillment costs decreases at the buyer level. Similar improvements are experienced at supplier while smoother demand patterns are realized. Setting, reviewing and maintaining performance goals, minimizing supply chain transactions through SKU’s, ensuring data accuracy, utilizing market intelligence to augment automated replenishment decisions, conducting performance reviews and using the metrics to find costs and inefficiencies, then eliminating them cooperatively are keys for successful VMI implementation.

As shorter product life cycles squeezed profit margins, manufacturers are forced to focus on cost-of-ownership and production-worthiness. As reviewed by Arnold [2] in a typical chip production facility, for every dollar worth of materials that stays in stock for a year, 35 cents are accounted for inventory expenses. Another article by Mahendroo [34] reviews the partnership between world’s leading semiconductor equipment manufacturing company Applied Materials and its customer, LSI. This partnership is an exemplary one in VMI context. Applied Material (AMAT) provides a service called Total Support Package to LSI to accelerate transition to its systems. As stated in AMAT’s annual report [1] Total Support Package covers all maintenance service and spare parts needed for Applied Material products, allowing LSI to quickly bring a system to production readiness without requiring additional investment in parts inventory build-up or adding/training new technical service support personnel. By monitoring and optimizing system performance on an on-going basis, this agreement reduced equipment operating costs, transaction costs by elimination of invoicing and accounts reconciliation, delivery costs through shipment consolidation, number of in house technicians and service part number duplication and administrative overhead costs while improving inventory standardization, management of inventories and service levels. Mahendroo [34] states that 15-30% lower cost and 200% tool utilization are obtained through this partnership.

A case study by Corbett et al. [20], presents the VMI relationship between Pelton International and its two customers: Perdielli Milan and Basco PLC. Pelton International is a multinational chemical firm. In that agreement, Pelton suggested consignment stocks as an incentive for standard keeping unit (SKU) rationalization to Perdielli and Basco. With that agreement Pelton international radically improved the relationship with Perdielli, increased standardization, reduced safety stocks and scheduling complexity, increased rationalization and reduced rush orders. On the customer side, Basco PLC exploited the benefits of consignment stock while experiencing more reliable deliveries related to integrated planning and forecasting. Perdielli Milan also reaped the benefits of consignment stock while reducing staff in purchasing department and got business experience in supply chain improvement which they began instituting with other suppliers. The relationship between Boeing, Rockwell Collins and Goodrich is another example for full service consignment that can be found in airframe maintenance sector [11]. The parts that are needed for airframe production is stored at customer sites or more commonly at Boeing warehouses in proximity to customer installations where logistics and transportation are handled by Boeing. The shift from traditional original equipment manufacturers to total service providers can be seen in this partnership.

Pan Pro LLC is a provider of advanced supply chain software solutions. In their web primer [36] they note the extensive information sharing and coordination requirement of VMI implementations. To achieve that, companies utilize technologies such as POS, EDI, XML, FTP and other reliable information sharing technologies. The level to which information will be shared and utilized are controlled by the contracts since information sharing certainly creates a strategic advantage which may be exploited by the partners in those contracts. It shall be ensured that both parties have strong incentives and commitment. VMI implementations will not be successful if required incentive, technical base and logistic infrastructure are not provided. Supply chains, which consist of multiple players with possibly conflicting objectives connected by flow of information, goods and money, often suffer from the quandary of conflicting performance measures. For example a low level of inventory may be a contradiction to high service level requirements. Contracts shall insure that parties will behave according to supply chain goals instead of their own goals. Obviously the nature of the products and demand affect how VMI will be implemented. For example in retail sector, inventory just enables the sales but as in our setting (capital equipment spare parts which consist of very expensive and critical material) inventory prevents unexpected and expensive down times and capacity losses. So the nature of the setting where VMI will be applied, shall be carefully integrated and contracts should be structured using this knowledge.

Other than participating to a consignment contract, the capital equipment manufacturer that we mentioned earlier also plans to jointly replenish the various locations in spare parts network. In existing practice, orders are treated separately, even if they come from various installations of the same customer. Under consignment contracts, the inventory control and decision rights of those locations are centralized under the control of the manufacturer which will allow the utilization of joint replenishment techniques. The Joint Replenishment Problem (JRP) has been a renowned research topic since it is a common real-world problem. JRP is also relevant when a group of items are purchased from the same supplier. The characteristics of the spare parts network such as multi product service requirement of the customers and existence of customers with multiple installations, are very similar to these two occurrences. By utilizing different modes of transportation, adjusting the timing and quantity of the replenishment, the manufacturer plans to exploit the benefits of JRP.

Before moving further, we explain how leadtimes and holding costs are improved under manufacturer control. As we mentioned before the spare parts that we are considering are very sensitive and high technology material which require special stocking environments and attention of expert personnel. The manufacturer has more technical expertise on the creating and maintaining such environments since she is the one who produces them. Also the manufacturer already has expert personnel for operating such environments. When retailer has to invest additional time and effort providing those requirements when she controls such environments. Therefore, we reflect this difference to costs in terms of holding costs. Also when manufacturer assumes the control, information systems of the manufacturer and the retailer are integrated. The stock rooms in retailer facilities are connected to the manufacturer's ERP software which provide continuous and precise monitoring. Consequently order processing times and invoicing activities are reduced which in turn reduces leadtimes. Other than that, the manufacturer utilizes different modes of transportation to replenish retailer facilities jointly which makes it easier to exploit benefits of mass transportation.

By utilizing consignment contracts and joint replenishment, the manufacturer aims to secure a market share by building strong relationships with its customers through contracts. Obviously being the preferred supplier of the majority of the customers in the market brings significant business advantages. Also with VMI and JRP, the manufacturer will obtain crucial demand data rapidly with less noise through integration of information systems which will in turn improve production plans, supply better coordination in deliveries and decrease ordering transactions. Obviously, the manufacturer wants to achieve short-term and long-term benefits that we specified in a profitable manner. All arrangements that are required to make VMI and JRP work, have costs significant costs, therefore this problem shall be carefully studied. In customers' perspective, in short term they will achieve increased product availability and backorder subsidies. In long term customers focus time and effort on their own operations rather than inventory management activities in return for an annual fee. Again profitability is the key for customer participation. When the whole supply chain is considered; elimination of incentive conflicts and provision of savings, which will be allocated to participants to improve their standings through utilization of VMI and JRP, are required to coordinate the channel. In this thesis, we first demonstrate the savings obtained from utilization of consignment contracts. By using the manufacturer's lower leadtime and holding cost, it is possible to achieve a lower total supply chain cost. Then we consider JRP and demonstrate that significant savings are possible by jointly replenishing multiple retailer installations that are part of a consignment contract. In various scenarios involving JRP and VMI, we investigate affect of various parameters such as holding costs, leadtime, ordering costs and backorder costs on these savings. By using this information, we search for the conditions (i.e. parameter ranges), under which parties agree to partnership. Obviously parties need to be better off than their initial standing to participate this contract. Finally we investigate how different allocation methods affect the participation and profits of the parties. We shall note that, even if one of the parties does not earn benefits from the contract, due to beforehand mentioned strategic reasons, she may choose to participate to contract. But in this research, we exclude that option.

The remainder of thesis is organized as follows. In Chapter 2, we provide a review of the literature in VMI, supply chain contracts, inventory theory and joint replenishment problem. In Chapter 3, we present the models for various inventory policies that will be used in investigating affects of VMI and JRP. Using those models, we construct contract models and formulate savings. In Chapter 4, we present our numerical results related to contracts without setup costs. We investigate supply chain coordinating values of various contract parameters. We also present savings achieved in supply chain through those contracts. In Chapter 5, we present the results of our numerical study related to contracts where there are setup costs. First effect of pure JRP will be demonstrated. Secondly the joint effect of VMI and JRP is demonstrated using comparison of $(Q, \mathbf{S})$ policy and (r, Q) policy. In Chapter 6, we conclude the thesis giving an overall summary of what we have done, our contribution to the existing literature and its practical implications.

Chapter 2

Literature Survey

Christopher [18] defines the supply chain as a network of organizations that are involved with upstream and downstream linkages in different processes and activities that produce value to the products or services. Persson [38] states the objectives of supply chain management as a set of cardinal beliefs; coordination and integration along the material flow, win-win relations and end customer focus. She also puts forward that there is much empirical evidence of benefits achieved when supply chain management is used effectively. For a long time the organizations in the supply chain have seen themselves as independent entities. But to survive in today's competitive environment, supply chains are becoming more integrated. First units of firms with similar functions become closer, then an internal integration occurs within the company and after that external integration with suppliers and customers occur. There are several concepts related to supply chain management and those are summarized by Waters [58] as follows:

- Improving communications: Integrated and increased communication within the supply chain with new technologies such as Electronic Data Interchange (EDI).
- Improving customer service: Increasing customer service levels while decreasing the costs.
- Globalization: As communication around the globe is increasing, companies become more international to survive in increasing competition and trade.

- Reduced number of suppliers: Better and long term relationships are created with a small number of suppliers.
- Concentration of ownership: Fewer players control the market.
- Outsourcing: Companies outsource more of their operations to 3^{rd} parties.
- Postponement: Goods are distributed to system in unfinished condition and final production is delayed.
- Cross-docking: Goods are directly shipped without being stored in warehouses.
- Direct delivery: The middle stages are eliminated and products are directly shipped from the manufacturer to the customer.
- Other stock reduction methods: Just-in-Time (JIT) and Vendor Managed Inventories (VMI) methods are employed.
- Increasing environmental concerns: Environmental considerations are gaining importance in logistics operations practices.
- Increasing collaboration along the supply chain: Objectives are unified and internal competition is eliminated within the supply chain.

In this research, results of several trends from above are investigated: improving customer service, globalization, employment of VMI methods and increasing collaboration along the supply chain through supply chain contracts.

Inventory systems have been extensively studied since the first half of the twentieth century. People from both industry and academy studied the subject in hope for attaining effective management of inventory using Operations Research tools. The most basic and critical questions: when to replenish and how much to replenish have been the focus of inventory management. Since inventory costs establish a significant portion of the costs that is faced by the firms, inventory management practices target maintaining a customer service level while holding the minimum possible amount of inventory. For example, Aschner [3] gives following five reasons for keeping inventories :

- Supply/Demand variations: Due to uncertainties in supplier performance and demand, safety stocks are kept.

- Anticipation: To meet seasonal demand, promotional demand and demand realized when production is unavailable, inventories are kept.
- Transportation: Due to high transportation leadtime and costs inventories are kept.
- Hedging: Considering price uncertainties (speculations, fluctuations or special opportunities), inventories are adjusted accordingly.
- Lot size: Replenishment amounts and leadtimes may not synchronize with the review period length and demand realization. Consequently inventories are adjusted accordingly.

Inventories may be classified in several ways. For example, Lambert [32] makes the following classification:

- Cycle stock: Inventory that is built because of the replenishment rules of relevant inventory policy.
- In-transit inventories: Material that is en-route from one location to another.
- Safety stock: Inventory that is held as an addition to cycle stock because demand uncertainty and order leadtime.
- Speculative stock: Inventory kept for reasons other than satisfying current demand.
- Seasonal stock: Inventory accumulated before a high demand season. This is a type of Speculative Stock.
- Dead stock: Items for which no demand has been realized for a time period.

Inventory theory has a well studied literature and it has been growing continually. Many old inventory models and policies are still used today. The classical Economic Order Quantity (EOQ) is used to calculate lot sizes when demand is deterministic and known for a single item. The approach is first suggested by Harris [29] but the model was published by Wilson [59]. In EOQ calculations, ordering and inventory holding costs are used to calculate optimal replenishment quantity. When demand is deterministic but varying over time in the former setting, optimal solution is calculated using the approach found by Wagner [56]. But this solution is using a clearly defined ending point and a backward perspective which decreases its applicability. Later, various heuristic methods are proposed and the most famous one is the Silver-Meal heuristic [44] since it is providing a solution with the lowest cost with forward perspective. Silver-Meal heuristic is also known as least period cost heuristic because of the forward perspective and it can work jointly with Material Requirements Planning (MRP) systems. Later, Baker [6] shows that Silver-Meal performs better than other heuristics in his review on the area.

In stochastic inventory theory literature, there are two types of models: Continuous review models and periodic review models. In continuous review models, the inventory position is monitored and updated continuously which implies that the inventory position changes are reflected to system instantly. In periodic review models, inventory position is reviewed and position changes are reflected to system periodically. Silver et al. [47] review four continuous review and periodic review models. First continuous review policy that is considered by Silver is the (r, Q) policy. When the inventory position reduces to the reorder point r, a fixed order quantity Q, which is calculated using EOQ formula, is ordered. The other continuous review policy that is considered is (s, S) policy which is placing an order of variable size to replenish the inventory to its order up to level as the inventory position is equal or below point s. In (r, Q) policy, size of the customer order is observed better. The base stock policy that we consider in this research, which is $(S - 1, S)$ policy, is a special case of (s, S) policy. This policy is generally used for items with relatively low demand and high cost, which perfectly suits our setting. For periodic review policies there are two widely used policies. The basic policy is the (r, R) policy where inventory position is inspected at every r units of time. At the time of inspection an order of variable type is placed to replenish the inventory to R. The next policy is the (r, s, R) policy. This policy is structured using (s, S) and (r, R) policies where $R = S$. At every r unit of time the inventory is checked but an order is only placed at the time of review if the inventory position at that time is in a higher place than s. In our research, we consider base-stock policy and (r, Q) policy for independently managed installations.

An echelon is a level in a supply chain and if a supply chain contains more than one level, it is called a multi-echelon inventory system. All inventory models that we presented until now were single-echelon systems. Now we will continue with multi-echelon inventory models, which consider chains consisting of several installations which keep inventories. Silver [47], Axsäter [5] and Zipkin [61] study this type of inventory systems. There are several ways to structure those systems:

- Series system: If two or more stocking points are linked. For example the first stocking point keeps the stock of a unfinished products and the second stocking point keeps the final product.
- Divergent distribution system: If each inventory location has at least one predecessor. A central distribution center serving to several retailers is an example.
- Convergent distribution system: If each inventory location has at least one immediate successor. An assembly system is an example.
- General systems: This type of systems can be any combination of formerly mentioned systems.

In our case, a divergent distribution system is investigated since there is one capital equipment manufacturing company which is serving more than one customers.

When there are multiple players in the supply chain, their activities need to be coordinated by a set of terms which is called a "supply chain contract". An important rationale for a contract is that it makes the relationship terms between parties explicit which enable parties to make realistic expectations and to identify legal obligations clearly. Generally, performance measures, such as delivery leadtimes, on-time delivery rates, and conformance rates are identified in contracts. These measures are used to quantify the performance of the relationship. There is a vast amount of literature on supply chain contracts. Two recent reviews of literature are Tsay et al. [51] and Cachon [10]. Tsay et al. provides an extensive review where they summarize model-based research on contracts in the various supply chain settings and provide an extensive literature survey of work in this area. Contracts may be structured using different concepts. Tsay et al. use the following classification [51]:

- specification of decision rights

- pricing
- minimum purchase commitments
- quantity flexibility
- buyback or returns policies
- allocation rules
- leadtimes
- quality

Cachon [10] reviews and extends the literature on management of incentive conflicts with contracts. In his work, he presents numerous supply chain models and for those he presents optimal supply chain actions and incentives for parties to comply to those actions. He reviews various contract types and presents benefits and drawbacks of each type. Here we review the supply chain contracting literature that is most relevant to our work: VMI and consignment contracts.

Fry et al. [22] introduce (Z, z) type of VMI contract which is proposed to bring savings due to better coordination of production and delivery. In this type of contract, the downstream party sets a minimum inventory level, z, and a maximum inventory level, Z, for her stock after realization of customer demand. The values of z and Z may represent explicit actual minimum and maximum levels of inventory or implicit values that are adjusted according to customer service levels and inventory turns. Downstream party charges upstream party a penalty cost if inventory level after realization of customer demand is larger or smaller than the contracted (Z, z) values. The optimal replenishment and production policies for supplier are found to be order-up-to policies. They compare this type of contract with classical Retailer Managed Inventory (RMI) with information sharing and find that it can perform significantly better than RMI in many settings but can perform worse in others. Corbett [19] studies incentive conflicts and information asymmetries in a multi-firm supply chain context using (r, Q) policy. He shows that traditional allocation of decision rights lead to inefficient solutions and he further analyzes the situation by considering two opposite situations. In the first case he presents the retailer's optimal menu of contracts, where supplier setup cost is unknown to buyer. Consignment stock is found to be helpful to reduce the impact of information asymmetries. In the second case, buyer's backorder cost is unknown to supplier and he presents that suppliers optimal menu of contracts on consignment stock. He finds that supplier has to overcompensate the buyer for the cost of each stock-out. According to Corbett, consignment stock helps reducing the cycle stock by providing additional incentive to decrease batch size but simultaneously gives the buyer an incentive to increase safety stock by exaggerating backorder costs.

Piplani and Viswanathan [39] study supplier owned inventory (SOI) which is an equivalent concept to consignment stock. They conduct a numerical study to investigate how various parameters affect the SOI contract and they find that as the ratio of buyer's demand to total demand of supplier increases, SOI agreements bring more savings to supply chain. They also note that as the ratio of supplier setup cost to buyer's ordering cost decreases, more savings are obtained. Wang et al. [57] shos that under a consignment contract, overall channel performance and individual performance of participants depend critically on demand price elasticity and the retailer's share of channel cost. They note that a consignment agreement naturally favors the retailer since she ties no money to inventory and she carries no risk. They model the contract process as a Stackelberg Game (leader-follower) where the retailer offers the contract to the manufacturer as a take-it-or-leave-it contract. Then the manufacturer participates if he can earn positive profit. They show that as price elasticity increases, channel performance degrades and as the retailer incurs more of the channel cost channel performance improves. Chaouch [15] investigates a VMI partnership under which supplier provides quicker replenishment. The model that is proposed is structured with the goal of finding the best trade-off among inventory investment, delivery rates considering some random demand pattern. The model also allows stock-outs. A solution is proposed which jointly determines delivery rates and stock levels that minimize transportation, inventory and shortage costs. Several numerical results are presented to give insight about the optimal policy's general behavior.

Choi et al. [17] study supplier performance under vendor managed inventory programs in capacitated supply chains. They show that supplier's service level is insufficient for the retailer to achieve desired service level at the customer end. How supplier achieves that service level, affects customer service level significantly. They provide a technique that considers lower bounds on customer service level, which takes average component shortage at supplier and stock out rate level into account. The contract they propose requires minimum amount of information sharing since it considers only demand distribution and the manufacturer capacity, which makes it easy, robust and flexible. We should note that this type of coordination is different from "transfer payment" methods.

Valentini and Zavanella [52] investigate how consignment stocks brings benefits and provided some managerial insights. They model the holding costs as two parts: storage part, which is classical holding cost, and financial part, which represents the opportunity costs that a firm incurs while investing financial resources in production. Using these costs, they model the inventories using (S, s) and (r, Q) policies. Fu and Piplani [23] study collaboration of between a supplier and the retailer by comparing two cases: the retailer makes inventory decisions with and without considering supplier's inventory policy. They show that collaboration has the ability to improve supply chain performance through better service levels and stabilizing effect. Lee and Schwarz [33] investigate three policies (periodic review policy, $(S, S-1)$ policy and (r, Q) policy) where a risk-neutral retailer delegates contract design to supplier whose hidden effort effects lead time. They show that supplier effort can change costs significantly and present the performance of optimal contracts they find under those policies.

We now review the literature on the joint replenishment problem. In an inventory system with multiple items or retailers, by coordination of replenishment of several items or retailers, cost savings can be obtained. Each time an order is placed, a major ordering cost is incurred, independent of the number of items ordered. Through jointly replenishing multiple retailers, companies aim to reduce the number of times that major ordering cost is charged which in turn decreases the total cost. Graves [27] discusses the similarities regarding cost functions and solutions procedures for the Joint Replenishment Problem, The Economic Lot Scheduling Problem (ELSP) and the One-warehouse N-retailer problem. Note that in terms of modeling there is no difference between multi-product, single installation models and single-product, multiple installation models. In the first case there are multiple items and a joint order is released when total demand to those items hit some threshold or an item's stock level is below its critical level, in the latter case same item is stocked in multiple locations and a joint order is released when total demand for that item hits the corresponding threshold or the stock level in an installation is below its critical level. This similarity is also addressed by Pantumsinchai [37].

The literature related to JRP consists of mainly two parts: deterministic demand and stochastic demand. For deterministic demand, indirect grouping strategies and direct grouping strategies are used. If an indirect grouping strategy is used, replenishment opportunities are considered at constant time intervals and order quantity of each item is selected in a way that it lasts for an integer multiple of the base time interval. Goyal introduces iterative methods in [24] and [26] to find the set of integer multiples of the base time interval by using an upper and lower bound for base time. He also presents an optimal solution in [25], which is giving the lowest possible cost, by improving the bounds on base time. In this paper he demonstrates that in general all optimal solutions and the most well performing heuristics are not simple policies. Most heuristics use the same underlying principle. First a time interval for the joint replenishments is found and then optimal order frequencies are determined. Then a new time interval is determined. This procedure is repeated until the solution converges. If direct grouping strategies are used, different items are grouped together to obtain better economies. For each group there is a base period time and all items within the group are replenished together. The challenging issue of direct grouping strategies is to divide the number of items into a certain number of different groups, since there can easily be a large amount of combinations to consider. Different algorithms of direct grouping that ranks the groups are presented by several authors. Firstly, Van Eijs [53] makes a comparison of direct and indirect grouping strategies on various setting. It is found that the indirect grouping methods produce lower cost solutions than direct grouping in scenarios where the major replenishment cost is large relative to the minor replenishment costs. Also Chakravarty's [13], [14] and Bastian's [8] works are crucial representatives of coordinated multi-item and/or multi-period inventory replenishment systems.

For stochastic demand case, the literature usually makes the following simplifying assumptions:

- Leadtimes are assumed to be deterministic or negligible.
- The entire order quantity is replenished at the same time.
- Holding costs for all items are at a constant rate per unit and unit time.
- There are no quantity discounts on the replenishments.
- The horizon is infinite.

In stochastic demand case, the JRP literature can be classified according to inventory policies that are used: continuous and periodic review policies. For continuous review systems, the most widely used policy in continuous review system is can-order policy, a.k.a $(\mathbf{S}, \mathbf{c}, \mathbf{s})$ policy. In this policy, system operates using three parameters: S_i, c_i and s_i for each item i. Note that $\mathbf{S}, \mathbf{c}, \mathbf{s}$ stands for a n-vectors such that $\mathbf{S}$=$(S_1, S_2, ..., S_n)$, $\mathbf{c}$=$(c_1, c_2, ..., c_n)$ and $\mathbf{s}$=$(s_1, s_2, ..., s_n)$ where n is number of items/installations. If inventory position of a particular item is below her individual s_i, a general replenishment order is triggered. In this replenishment all items with inventory positions less than their individual c_i level, are replenished up to their individual S_i level. This policy is first proposed by Balintfy [7] and he called it the random joint order policy. Balintfy investigates the case that the demand distribution is negative exponential. Then Silver [43] investigates the case where there are two items having identical cost and Poisson demand. Later Ignall [31] examines the same problem where there are two independent Poisson demands. Silver [44] extends the content and studies three different methods and obtains the same total cost function of the problem under Poisson demand and with zero leadtimes. Silver [45] broadens his study over constant leadtimes. He also shows that it is possible to have significant cost savings using $(\mathbf{S}, \mathbf{c}, \mathbf{s})$ policy instead of individual ordering policies. Later, Silver and Thompstone [50] consider a setting where demand is compound Poisson with zero leadtime and find closed form cost expressions for this setting. Under compound Poisson demand and non-zero leadtimes; Shaack [41], Silver [46], Federgruen et al. [21], Schultz [42] and Melchiors [35] suggest different methods to find control variables. Federgruen et al. [21] study a continuous review multi-item inventory system in which demands follow an independent compound Poisson process. An efficient heuristic algorithm to search for an optimal rule is proposed where numerical analysis show that the algorithm performs slightly better than the heuristic of Silver and can handle nonzero leadtimes and compound Poisson demand. Moreover, it is seen that significant cost savings can be achieved by using the suboptimal coordinated control instead of individual control. We should note that much of the research is focused on the $(\mathbf{S}, \mathbf{c}, \mathbf{s})$ policies.

First author to study periodic inventory review policies in JRP literature is Sivazlian [48]. He proposes mixed ordering policies. In this type of policies; zero, one or multiple items may be ordered at the time of replenishment. Two replenishment policies are proposed by Atkins and Iyogun [4]. First one is a periodic policy where all items are ordered up to the base stock level at every replenishment time. Second one is modified periodic review policy where a core set of items are replenished at every replenishment instance and remaining items are replenished at specific replenishment instances. His modified periodic policy performs better than the $(\mathbf{S}, \mathbf{c}, \mathbf{s})$ policy in some cases. Cheung and Lee [16] study the effects of coordinated replenishments and stock rebalancing. With shipment coordination, the ordering decisions of retailers are done by the supplier using the information that the retailers provide to the supplier. Stock rebalancing is used to rebalance retailers' inventory positions. Analysis of shipment coordination is useful in the sense that, it can be used for joint replenishment analysis. Instead of n retailers, we can consider n items (due to the fact that the authors use the same leadtime for all retailers here). Cheung and Lee consider a policy such that the demand for the total of n retailers reach to Q, a replenishment order is made. A similar policy is better presented in Pantumsinchai's paper [37].

Çetinkaya and Lee [12] presents an analytical model to coordinate the inventory and transportation decisions of the supply chain. Instead of immediately delivering the orders, the supplier waits for a time period to consolidate the orders coming from different retailers to coordinate shipments. The problem is finding the replenishment quantity and dispatch frequency that will minimize the cost of the system. A time-based consolidation policy is used and it is found that this policy can outperform classical policies under some conditions.

Balintfy [7] compares the individual order policy, the joint order policy, where a setup cost reduction is possible by jointly ordering the items, and the random ordering policy, which is in between joint and individual ordering policies. In this paper he gives some easy to compare results to determine which policy to use in which instances. Moreover, it is shown that the random joint ordering policy is always better that individual ordering policy.

Pantumsinchai [37] extends the $(Q, \mathbf{S})$ policy for Poisson demands. This policy tracks the total usage of several items since the last replenishment and if that amount passes a threshold, all items are replenished up to their base stock level. This model is originally studied by Renberg [40]. It outperforms $(\mathbf{S}, \mathbf{c}, \mathbf{s})$ policy when there is a small number of items with similar demand pattern and high ordering cost. Viswanathan [54] studies $P(\mathbf{s}, \mathbf{S})$ policy which is applying an individual (s_i, S_i) policy to all items at every review period. Every item with inventory position below their individual s, is included in the replenishment. In his paper, he shows that $P(\mathbf{s}, \mathbf{S})$ policy is proved to outperform earlier approaches most of the test cases. Later he studies optimal algorithms for the joint replenishment problem in his work [55]. Cachon [9] studies three dispatch policies (a minimum quantity continuous review policy, a full service periodic review policy, and a minimum quantity periodic review policy) where truck capacity is finite, a fixed shipping and per unit shelf-space cost is incurred. In the numerical study he finds that either of the two periodic review policies may have substantially higher costs than the continuous review policy especially when leadtime is short. In that case EOQ heuristic performs quite well.

We note that the primary difference between our study and earlier research is that we extend the consignment contracts literature in the direction of joint replenishment. We consider savings brought by physical improvement and joint replenishment simultaneously in a consignment contract for the first time. We use backorder costs and the annual fee as the terms of the contract and search for values of these variables which coordinate the supply chain.

Chapter 3

Models

We consider an inventory system which consists of a manufacturer and a retailer (perhaps with multiple installations). We first model a single retailer installation which does not have any setup costs and uses a base stock policy. For this case, we study a consignment contract, under which the manufacturer takes the ownership and the responsibility of the inventory. Since there are no setup costs, the manufacturer also uses a base stock policy. In the second case, there are multiple retailer installations and there are setup costs for ordering. Before the contract, the retailer manages its installations independently using an (r, Q) policy. After the contract, the manufacturer manages the inventories of multiple installations jointly using a $(Q, \mathbf{S})$ policy. We first review base stock policy, (r, Q) policy and $(Q, \mathbf{S})$ policy models and then explain the setup before and after the contract.

We now present common assumptions and notation that are used in all models. We assume the following.

- Demands arrive according to a Poisson Process,
- Size of each demand is discrete and equals to 1,
- Leadtimes are deterministic,
- Policy variables such as base stock levels, reorder levels and order quantities are discrete,

Notation:

λ = Arrival rate per time,
L = Replenishment leadtime,
S = Base stock level,
r = Reorder level,
Q = Reorder quantity,
h = Holding cost,
K = Setup cost,
π = Backorder cost per occasion (type I backorder),
π' = Backorder cost per unit per time (type II backorder),
BO_1 = Type I per occasion backorder cost term,
BO_2 = Type II per unit per time backorder cost term,

We use (r, Q) and base stock policies as explained in Hadley and Whitin [28]. $(Q, \mathbf{S})$ model defined by Pantumsinchai [37] is used where minor setup costs are neglected. This $(Q, \mathbf{S})$ model is also similar to the model by Cachon [9] but without capacity constraints.

There is a common ordering cost K which is charged every time a replenishment order is placed. It is related with transportation/ordering costs and is independent of number of items involved in the order. Holding cost h is charged per unit item kept in the inventory per unit time. Type I backorder cost, π, is charged for each stockout occasion and Type II backorder cost, π', is charged for each backordered unit per time. In each policy, the objective is to minimize expected total cost per unit time. Inventory position is calculated as on hand inventory plus on order inventory minus backorders.

3.1 Base Stock Policy

We use base stock policy to model the inventory of an individual customer installation when there is no setup cost. In the base stock policy, a discrete order up to level, S, is determined. Inventory is reviewed continuously and as soon as a demand is realized, an order is issued. Therefore the inventory position is equal to S at all times. This policy is also known as $(S-1, S)$ policy, or one-for-one policy.

Now consider an arbitrary time t. If there was no demand between $t-L$ and t, the on hand inventory would be equal to S, since all replenishment orders that were placed before $t-L$ would be received by time t. Therefore, the inventory on hand and the amount of backorders at time t only depend on the demand that is realized between $t-L$ and t, i.e., demand during lead time.

Poisson probability of observing x unit demands during lead time is given by

$$p(x, \lambda L) = \frac{e^{-\lambda L}(\lambda L)^x}{x!}. \tag{3.1}$$

Therefore, Poisson probability of observing x or more demands during in lead time is given by

$$P(x, \lambda L) = \sum_{z=x}^{\infty} p(z, \lambda L). \tag{3.2}$$

Now, if there are $S-y$ demands $(0 \leq y < S)$ that are realized during lead time, then the inventory on hand at time t would be y. If there are S or more than S demands that are realized during lead time, then the inventory on hand at time t would be 0. Therefore, the probability of having y units on hand at an arbitrary time t is given by,

$$\psi_1(y) = \begin{cases} p(S-y, \lambda L) & \text{if } \ 0 < y \leq S \\ P(S, \lambda L) & \text{if } \ y = 0. \end{cases} \tag{3.3}$$

Similarly, if there are $S+y$ demands $(y \geq 0)$ that are realized during lead time, then the amount of backorders at time t is y. Therefore, the probability of having y backorders at any arbitrary time t can be written as

$$\psi_2(y) = \ p(S+y, \lambda L) \quad \text{where } y \geq 0. \tag{3.4}$$

Then, the probability of being in an out of stock state at any arbitrary time t is given as

$$P_{out} = \sum_{y=0}^{\infty} \ \psi_2(y) = \mathrm{P}(\mathrm{S}, \ \lambda L). \tag{3.5}$$

Therefore, the average number of backorders per unit of time is given by

$$E(S) = \lambda P_{out}. \tag{3.6}$$

Similarly, the expected number of backorders at any arbitrary time t can be written as

$$B(S) = \sum_{y=0}^{\infty} y\psi_2(y). \tag{3.7}$$

Expected on hand inventory at any arbitrary time t can be written as

$$\chi(S) = S - \lambda L + B(S). \tag{3.8}$$

Finally, the total cost of the installation under base stock policy can be written as

$$\Omega(S) = h\chi(S) + \pi E(S) + \pi' B(S). \tag{3.9}$$

3.2 (r, Q) Policy

We use the (r, Q) policy as discussed in Hadley and Within [28] to model the inventory of an individual retailer installation when there are setup costs. In this model, the reorder level, r, the reorder quantity, Q, and all other inventory levels are discrete and positive integers. Again unit Poisson demands are assumed. When inventory position falls below r, an order of magnitude Q is immediately placed so that the inventory position raises to $r + Q$ after the order. Inventory position must have one of the values $r + 1$, $r + 2$,...,$r + Q$. It is never in inventory position r for a finite length of time. It can be shown that each of inventory position, $r + j$ has a probability $\rho(r + j) = \frac{1}{Q}$ for $j = 1, ..., Q$ [28].

Inventory position, by itself, does not tell us anything about the on hand inventory or the net inventory. If the inventory position is $r + j$, there may be no orders outstanding with the net inventory being $r + j$ or one order outstanding with net inventory being $r+j-Q$. For Poisson demands, where there is a positive probability for an arbitrarily large quantity being demanded in any time interval, it is theoretically possible to have any number of orders outstanding at a particular instant of time.

The probability of having y items on hand at any arbitrary time t can be written as

$$\begin{aligned} \psi_1(x) &= \tfrac{1}{Q}\textstyle\sum_{j=1}^{Q} p(r+j-y,\lambda L) \\ &= \tfrac{1}{Q}[1-P(r+Q+1-x,\lambda L),] \text{ where } r+1 \leq x \leq r+Q. \end{aligned} \tag{3.10}$$

The probability of having y backorders at any arbitrary time t can be given as

$$\begin{aligned} \psi_2(y) &= \tfrac{1}{Q}\textstyle\sum_{j=1}^{Q} p(r+y+j,\lambda L) \\ &= \tfrac{1}{Q}[P(r+y+1,\lambda L)-P(r+y+Q+1,\lambda L)], \text{ where } y \geq 0. \end{aligned} \tag{3.11}$$

Then, the probability of being in an out of stock state at any arbitrary time t can be written as

$$\begin{aligned} P_{out} &= \textstyle\sum_{y=0}^{\infty} \psi_2(y) \\ &= \tfrac{1}{Q}[\textstyle\sum_{u=r+1}^{\infty} P(u,\lambda L) - \sum_{u=r+Q+1}^{\infty} P(u,\lambda L)]. \end{aligned} \tag{3.12}$$

Therefore, the average number of backorders per unit of time can be given as

$$E(Q,r) = \lambda P_{out}. \tag{3.13}$$

The expected number of backorders at any arbitrary time t can be given as

$$\begin{aligned} B(Q,r) &= \textstyle\sum_{y=0}^{\infty} y\psi_2(y) \\ &= \tfrac{1}{Q}[\textstyle\sum_{u=r+1}^{\infty} P(u-r-1,\lambda L) - \sum_{u=r+Q+1}^{\infty} P(u-r-Q-1,\lambda L)]. \end{aligned} \tag{3.14}$$

The expected on hand inventory at any arbitrary time t can be written as

$$\begin{aligned} \chi(Q,r) &= \textstyle\sum_{x=0}^{r+Q} x\psi_1(x) \\ &= \tfrac{Q+1}{2} + r - \lambda L + B(Q,r). \end{aligned} \tag{3.15}$$

Finally, the expected total cost rate of an installation under (r,Q) policy can be formulated as

$$\Omega(Q,r) = K\frac{\lambda}{Q} + h\chi(Q,r) + \pi E(Q,r) + \pi' B(Q,r). \tag{3.16}$$

3.3 $(Q, \mathbf{S})$ Policy

In this section, we model inventories of n installations of a retailer using $(Q, \mathbf{S})$ policy introduced by Renberg and Planche [40]. Pantumsinchai [37] characterized this policy under Poisson demands. In this model, each installation i has a base stock level, S^i and for the whole system, there is an order quantity, Q. Demand is realized by each retailer according to a Poisson process with rate λ^i. All unmet demands are assumed to be backordered. Each retailer installation has a leadtime, L_i and system is under continuous review. Assume for the simplicity of the exposition that the holding cost and backorder cost parameters are same, i.e., $h^i = h$, $\pi^i = \pi$, and $\pi'^i = \pi'$ for all i. Information about the last replenishment, the time elapsed since the last replenishment and the demand realized since last replenishment is available. As soon as Q total demands are realized since the last order, a new order is released. In the system, total inventory position of all retailers is denoted by, $S = \sum_{i=1}^{n} S^i$. When the demand realized by n installations accumulates to Q, inventory position drops to "group reorder point" which is equal to $s = S - Q$. When an order is placed, a new cycle is initiated.

Combined arrival rate to system is given by

$$\lambda = \sum_{i=1}^{n} \lambda^i. \tag{3.17}$$

Poisson probability of installation i facing a demand of size d^i during leadtime can be written as

$$r^i(d^i) = \frac{e^{-\lambda^i L^i}(\lambda^i L^i)^{d^i}}{d^i!} \quad \forall\, d^i \geq 0. \tag{3.18}$$

Let the demand realized by installation i since last order be x^i. Then the inventory position of installation i since last order can be written as

$$z^i = S^i - x^i \quad \forall i = 1, ..., n. \tag{3.19}$$

Thus, the combined inventory position of the system since last order can be written as

$$z = \sum_{i=1}^{n} z^i. \tag{3.20}$$

Finally, the total demand realized in the system since last order can be written as

$$x = \sum_{i=1}^{n} x^i = \sum_{i=1}^{n} S^i - \sum_{i=1}^{n} z^i = S - z. \tag{3.21}$$

Under the $(Q, \mathbf{S})$ policy, an installation inventory position follows a regenerative process and has a steady state distribution. For simple Poisson Process, the conditional probability $P(x^i|x)$ is binomial with parameters x and λ^i/λ. Steady state distribution of x, is uniform between 0 and $Q-1$ as given in Hadley and Whitin [28]. Equivalently z is uniformly distributed between S and s. Hence, the marginal distribution of x^i, $u^i(x^i)$, can be derived as

$$u^i(x^i) = \frac{1}{Q} \sum_{x=x^i}^{Q-1} \binom{x}{x^i} (\lambda^i/\lambda)^{x^i} (1-\lambda^i/\lambda)^{x-x^i} \quad x^i = 0, 1, ..., Q-1. \tag{3.22}$$

Pantumsinchai [37] shows that this distribution is equivalent to

$$u^i(x^i) = \frac{\lambda}{\lambda^i Q}(1 - B^i(x^i, Q, \lambda^i/\lambda)) \quad x^i = 0, 1, ..., Q-1. \tag{3.23}$$

where $B^i(x^i, Q, \lambda^i/\lambda)$ is the cumulative binomial probability.

Then the net inventory of installation i in steady state becomes $S^i - x^i - d^i = S^i - v^i$ where v^i is a random variable with probability distribution $m^i(v^i)$:

$$m^i(v^i) = \sum_{x^i=0}^{min(v^i, Q-1)} u^i(x^i) r^i(v^i - x^i) \quad v^i = 0, 1, 2, ... \tag{3.24}$$

The stock-out probability of installation i at any arbitrary time t can be written as

$$P^i(S^i, Q^i) = \Pr(v^i \geq s^i) = \sum_{v^i=S^i}^{\infty} m^i(v^i). \tag{3.25}$$

The expected size of backorder at installation i at any arbitrary time t can be formulated as

$$B^i(S^i, Q^i) = \sum_{v^i=S^i+1}^{\infty} (v^i - S^i) m^i(v^i). \tag{3.26}$$

Then, the expected number of items in stock out condition at installation i at any arbitrary time can be given as

$$\sum_{i=1}^{n} P^i(S^i, Q). \tag{3.27}$$

The expected inventory on hand at installation i at any arbitrary time can be given as

$$\chi^i(S^i, Q) = S^i - \tfrac{(Q-1)\lambda^i}{2\lambda} - \lambda L^i + B^i(S^i, Q^i). \tag{3.28}$$

The safety stock at installation i at any arbitrary time can be given as

$$S^i - (\lambda^i/\lambda)Q - \lambda^i L^i. \tag{3.29}$$

Also note that probability that installation i will not contribute an order can be formulated as

$$u^i(0) = (1 - \lambda^i/\lambda)^Q. \tag{3.30}$$

Now let us denote the vector that contains base stock levels of n installations as $\mathbf{S}$ such that $\mathbf{S}= (S^1, S^2, ..., S^n)$.

The total cost rate of n installations under $(Q, \mathbf{S})$ policy can be formulated as

$$\begin{aligned} \Omega(Q, \mathbf{S}) \;&= K\tfrac{\lambda}{Q} + h\textstyle\sum_{i=1}^n \chi^i(S^i, Q) + \sum_{i=1}^n \pi' B^i(S^i, Q) + \sum_{i=1}^n \pi\lambda^i P^i(S^i, Q) \\ &= K\tfrac{\lambda}{Q} + \textstyle\sum_{i=1}^n h(S^i - \tfrac{\lambda^i(Q-1)}{2\lambda} - \lambda^i L^i) + \sum_{i=1}^n (\pi' + h)B^i(S^i, Q) \\ &+ \textstyle\sum_{i=1}^n \pi\lambda^i P^i(S^i, Q). \end{aligned} \tag{3.31}$$

First three terms of the cost function is convex in Q and S^i. Zipkin [60] shows that $B^i(S^i, Q)$ is convex in S^is and Q and jointly in S^is and Q when $n = 1$. $P^i(S^i, Q)$ is also shown to be convex under nonnegative safety stock assumption. It is also shown that,

If $\pi = 0$, cost function is strictly convex in S.
If $\pi \geq 0$, cost function is convex in S when $m^i(v^i)$ is monotonically decreasing.

Finally it is shown that when $L = 0$, $m(\cdot)$ is equivalent to $u(\cdot)$.

In order to find the locally optimal values Q^* and $\mathbf{S}^*$ of Q and $\mathbf{S}$, we use the following algorithm used by Pantumsinchai [37]. First the initial value of Q is set to $Q_0 = \max\{1, \sqrt{\frac{2\lambda K}{\sum_{i=1}^n (\lambda^i/\lambda)h}}\}$. For this given value of $Q = Q_0$, new values of Q is searched inside the range $[\max\{0, Q_0 - M\}, Q_0 + M]$. With all values of Q inside this range, the corresponding values of S^i need to be found. For a given Q and for each i, the new value of S^i is the smallest integer that satisfies

$$(\pi' + h)\sum_{v^i=0}^{S^i} m^i(v^i) - \pi\lambda^i m^i(S^i) \geq \pi'. \tag{3.32}$$

Or more formally, $S^i_0 = \min\{S^i : \gamma^i(S^i, Q) \geq \pi'\}$ where

$$\gamma^i(S^i, Q) = (\pi' + h)\sum_{v^i=0}^{S^i} m^i(v^i) - \pi\lambda^i m^i(S^i) \tag{3.33}$$

Note that the function m^i above is also a function of Q. With each value of Q and corresponding S^i values, the objective function $\Omega(Q, S^1, S^2, ..., S^n)$ is evaluated. The Q value that gives the minimum objective function value is taken as the new value of Q, and a new iteration starts. The algorithm stops at iteration k with $Q^* = Q_k$ (and corresponding S^{i*} found using 3.32) when none of the Q values in the range $[\max\{0, Q_k - M\}, Q_k + M]$ gives a lower objective function value. Using larger values of M will increase the chances of finding the global optimum, but will slow down the algorithm. Following Pantumsinchai [37], we use $M = 20$. This algorithm is more formally defined in Algorithm 1.

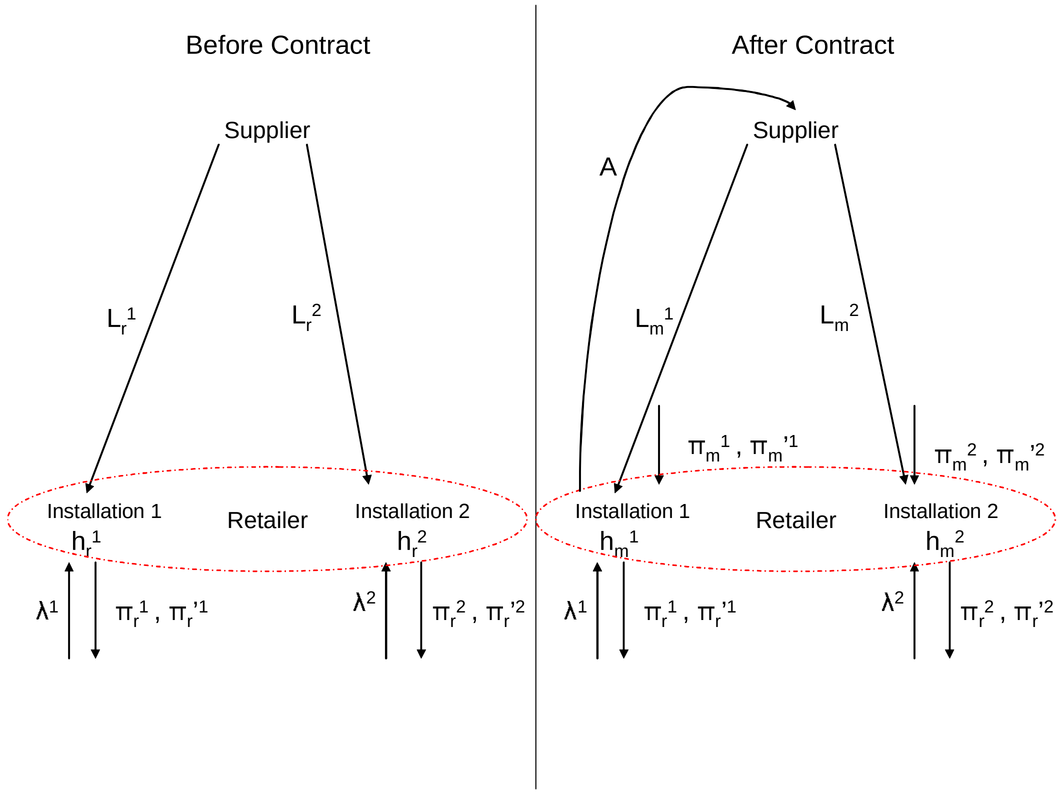
3.4 Contracts

In this section, we structure the contracts using models we previously defined. Without loss of generality, we call upstream location on the supply chain as "manufacturer" and downstream location as "retailer". In Figure 3.1, we depict the change in parameters when the manufacturer assumes the control, after the contract.

Algorithm 1 Algorithm for finding locally optimal Q and S values

Set $M := 20$
Set $Q_0 := \max\{1, \sqrt{\frac{2\lambda K}{\sum_{i=1}^{n}(\lambda^i/\lambda)h}}\}$
Set $S_0^i := \min\{S^i : \gamma^i(S^i, Q_0) > \pi'\}$ for each i
Set $\Omega^0 := \Omega(Q_0, S_0^1, S_0^2, ..., S_0^n)$
Set $k := 0$
repeat
  Set $k := k + 1$
  Set $Q_k := Q_{k-1}$
  Set $S_k^i := S_{k-1}^i$ for each i
  Set $\Omega_k := \Omega_{k-1}$
  for Q^{temp}:=max$\{0, Q_{k-1} - M\}...Q_{k-1} + M$ **do**
    Set $S_{temp}^i := \min\{S^i : \gamma^i(S^i, Q_{temp}) > \pi'\}$ for each i
    Set $\Omega_{temp} := \Omega(Q_{temp}, S_{temp}^1, S_{temp}^2, ..., S_{temp}^n)$
    if $\Omega_{temp} < \Omega_k$ **then**
      Set $\Omega_k := \Omega_{temp}$
      Set $Q_k := Q_{temp}$
      Set $S_k^i := S_{temp}^i$ for each i
    end if
  end for
until $\Omega_k \geq \Omega_{k-1}$
Set $Q^* := Q_k$
Set $S^{i*} := S_k^i$ for each i

Figure 3.1: Supply Chain Parameters Before and After Contract



Additional notation used in this section is as following:

λ^i = Demand arrival rate per time at each installation i,
λ = Combined arrival rate per time,
L_r^i = Retailer's replenishment leadtime for installation i,
L_m^i = Manufacturer's replenishment leadtime for installation i,
S_r^i = Base stock level optimizing total cost rate of installation i under the retailer control,
S_m^i = Base stock level optimizing total cost rate of installation i under the manufacturer control,
r_r^i = Reorder level optimizing total cost rate of installation i under the retailer control,
Q_r^i = Reorder quantity optimizing total cost rate of installation i under the retailer control,
Q_m = Reorder quantity optimizing total cost rate system under manufacturer control,
h_r = Holding cost per unit per time for the retailer,
h_m = Holding cost per unit per time for the manufacturer,
K = Setup cost for ordering,
π_r = Backorder cost per occasion observed by the retailer,
π'_r = Backorder cost per unit per time observed by the retailer,
π_m = Backorder cost per occasion charged by the retailer to the manufacturer,
π'_m = Backorder cost per unit per time charged by the retailer to the manufacturer,
Ω_r = Total expected cost rate of the retailer before contract
Ω_m = Total expected cost rate of the manufacturer before contract
Ω_{sc} = Total expected cost rate of the supply chain before contract
Ω_r^c = Total expected cost rate of the retailer after contract
Ω_m^c = Total expected cost rate of the manufacturer after contract
Ω_{sc}^c = Total expected cost rate of the supply chain after contract
A = Annual fee paid by the retailer to the manufacturer for the contract.

3.4.1 Without Setup Costs

First we consider the case with no setup costs. Using the base stock model we derive the total costs of the retailer and the manufacturer. Before the contract, the retailer manages her own inventory according to her own cost parameters. Supply chain cost rate, Ω_{sc}, is equal to the retailer's cost rate, Ω_r. These costs are given below:

$$\Omega_r(S) = h_r\chi(S, L_r) + \pi_r E(S, L_r) + \pi'_r B(S, L_r) \tag{3.34}$$
$$\Omega_m(S) = 0 \tag{3.35}$$
$$\Omega_{sc}(S) = h_r\chi(S, L_r) + \pi_r E(S, L_r) + \pi'_r B(S, L_r). \tag{3.36}$$

Let S_r is the base stock level optimizing retailers total cost rate,

$$S_r = \arg\min \Omega_r(S). \tag{3.37}$$

After the consignment contract, the manufacturer assumes the control of inventory. In this case, the manufacturer has an improved leadtime, $L_m \leq L_r$, and holding cost per unit per time $h_m \leq h_r$. Using these parameters and the backorder costs incurred by the retailer, π_m and π'_m, the manufacturer optimizes Ω^c_m.

The annual fee payed by retailer to manufacturer is, A. After contract:

$$\Omega^c_r(S) = (\pi_r - \pi_m)E(S, L_m) + (\pi'_r - \pi'_m)B(S, L_m) + A. \tag{3.38}$$
$$\Omega^c_m(S) = h_m\chi(S, L_m) + \pi_m E(S, L_m) + \pi'_m B(S, L_m) - A. \tag{3.39}$$
$$\Omega^c_{sc}(S) = h_m\chi(S, L_m) + \pi_r E(S, L_m) + \pi'_r B(S, L_m). \tag{3.40}$$

S_m is the base stock level optimizing the manufacturer's after contract total cost rate,

$$S_m = \arg\min \Omega^c_m(S). \tag{3.41}$$

Supply chain saving that is achieved by the implementation of the contract can be given as:

$$\begin{aligned} &= \Omega^c_{sc}(S_m) - \Omega_{sc}(S_r) \\ &= (h_m\chi(S_m, L_m) + \pi_r E(S_m, L_m) + \pi'_r B(S_m, L_m)) \\ &- (h_r\ \chi(S_r,\ L_r) + \pi_r\ E(S_r,\ L_r) + \pi'_r\ B(S_r,\ L_r)). \end{aligned} \tag{3.42}$$

Note that the supply chain costs are minimized (or the savings are maximized), i.e., the channel is coordinated, only if the retailer charges the same backorder penalties that she observes, i.e., $\pi_m = \pi_r$ and $\pi'_m = \pi'_r$. Because, only in this case, the manufacturer (who makes the decision on S) and the supply chain have the same cost function (i.e., objective function) with the exclusion of the fixed payment A which does not depend on S.

For the retailer and manufacturer to participate in the contract, both have to be better off with the contract. Thus, the following conditions should be satisfied.

$$\Omega_m(S_r) \geq \Omega^c_m(S_m). \tag{3.43}$$

$$\Omega_r(S_r) \geq \Omega^c_r(S_m). \tag{3.44}$$

These two conditions enforce upper and lower bound constraints on A. If those two conditions are satisfied, the contract is possible:

$$A \geq h_m \chi(S_m, L_m) + \pi_m E(S_m, L_m) + \pi'_m B(S_m, L_m) \tag{3.45}$$

$$\begin{aligned} A \leq {} & (h_r\ \chi(S_r,\ L_r) + \pi_r\ E(S_r,\ L_r) + \pi'_r\ B(S_r,\ L_r)) \\ & -(\pi_r - \pi_m)E(S_m, L_m) - (\pi'_r - \pi'_m)B(S_m, L_m). \end{aligned} \tag{3.46}$$

Note finally that a feasible A can be found, only if the supply chain cost savings are non-negative. The exact value of A that is used in the contract specifies how the savings through the contract are allocated to both parties. The backorder penalties charged by the retailer to the manufacturer also impact the final costs of each party and thus the allocation of total supply chain costs. However, as discussed before, backorder penalties that are different from the original backorder penalties result in a non–coordinated channel, and thus should not be used as an allocation mechanism.

3.4.2 With Setup Costs

Using (r, Q) and $(Q, \mathbf{S})$ models we derive the total costs of retailer and manufacturer when there are setup costs. Initially each retailer installation use (r, Q) model to manage their inventories. We assume that there are n installations. Let us define the following n-vectors for simplicity. First one contains individual ordering quantities of retailer installations $\mathbf{Q}= (Q^1, Q^2, ..., Q^n)$. Second one contains individual reorder levels of retailer installations $\mathbf{r}=(r^1, r^2, ..., r^n)$. Before the contract the total costs of eacc party for a given $\mathbf{Q}$ and $\mathbf{r}$ can be written as:

$$\Omega_r^i(Q^i, r^i) = K\frac{\lambda^i}{Q^i} + h_r\chi(Q^i, r^i, L_r^i) + \pi_r E(Q^i, r^i, L_r^i) + \pi_r' B(Q^i, r^i, L_r^i) \tag{3.47}$$

$$\Omega_r(\mathbf{Q}, \mathbf{r}) = \sum_{i=1}^{n} \Omega_r^i(Q^i, r^i) \tag{3.48}$$

$$\Omega_m(\mathbf{Q}, \mathbf{r}) = 0. \tag{3.49}$$

$$\Omega_{sc}(\mathbf{Q}, \mathbf{r}) = \sum_{i=1}^{n} \Omega_r^i(Q^i, r^i). \tag{3.50}$$

For installation i, Let r_r^i and Q_r^i denote the reorder quantity and reorder level that minimizes installation i's total cost rate, Ω_r^i. Formally,

$$(Q_r^i, r_r^i) = \arg\min \Omega_r^i(Q^i, r^i) \quad \text{for} \quad i = 1, ..., n. \tag{3.51}$$

Then, $\mathbf{Q_r}=(Q_r^1, Q_r^2, ..., Q_r^n)$ and $\mathbf{r_r}=(r_r^1, r_r^2, ..., r_r^n)$

The supply chain cost rate, and the total retailer cost rate are equal to the sum of cost rates of installations, i.e.,

$$\Omega_{sc}(\mathbf{Q_r}, \mathbf{r_r}) = \Omega_r(\mathbf{Q_r}, \mathbf{r_r}) = \sum_{i=1}^{n} \Omega_r^i(Q_r^i, r_r^i). \tag{3.52}$$

After the consignment contract, the manufacturer assumes the control of inventory. She starts to use $(Q, \mathbf{S})$ policy to jointly replenish installations. In this case manufacturer has an improved leadtime, $L_m^i \leq L_r^i$, setup cost and holding cost, $h_m \leq h_r$. With these parameters, the backorder costs incurred by retailer, π_m and π_m' and for a given $(Q, \mathbf{S})$, the cost of each party after the contract can be written as:

$$\Omega_r^c(Q,\mathbf{S}) = \sum_{i=1}^{n}(\pi_r - \pi_m)\lambda^i P^i(S^i,Q,L_m^i) + \sum_{i=1}^{n}(\pi_r' - \pi_m')B^i(S^i,Q,L_m^i) + A \quad (3.53)$$

$$\Omega_m^c(Q,\mathbf{S}) = K\frac{\lambda}{Q} + \sum_{i=1}^{n}(h_m)(S^i - \frac{\lambda_i(Q-1)}{2\lambda} - \lambda^i L_m^i) + \quad (3.54)$$

$$\sum_{i=1}^{n}(\pi_m' + h_m)B^i(S^i,Q,L_m^i) +$$

$$\sum_{i=1}^{n}\pi_m P^i(S^i,Q,L_m^i) - A$$

$$\Omega_{sc}^c(Q,\mathbf{S}) = K\frac{\lambda}{Q} + \sum_{i=1}^{n}(h_m)(S^i - \frac{\lambda^i(Q-1)}{2\lambda} - \lambda^i L_m^i) + \quad (3.55)$$

$$\sum_{i=1}^{n}(\pi_r' + h_m)B^i(S^i,Q,L_m^i) +$$

$$\sum_{i=1}^{n}\pi_r\lambda^i P^i(S^i,Q,L_m^i).$$

Let Q_m be the optimal joint ordering quantity and S_m^i be the optimal base stock level of each installation. Let $\mathbf{S_m}$= $(S_m^1, S_m^2, ..., S_m^n)$. Formally,

$$(Q_m, \mathbf{S_m}) = \arg\min \Omega_m^c(Q,\mathbf{S}). \quad (3.56)$$

Supply chain saving that is achieved by the implementation of the contract can be given as:

$$\Omega_{sc}(\mathbf{Q_r}, \mathbf{r_r}) - \Omega_{sc}^c(Q_m, \mathbf{S_m}) \quad (3.57)$$

$$= \sum_{i=1}^{n}\left[K\frac{\lambda^i}{Q_r^i} + h_r\chi(Q_r^i, r_r^i, L_r^i) + \pi_r E(Q_r^i, r_r^i, L_r^i) + \pi_r' B(Q_r^i, r_r^i, L_r^i)\right]$$

$$- K\frac{\lambda}{Q_m} + \sum_{i=1}^{n}(h_m)(S_m^i - \frac{\lambda^i(Q_m-1)}{2\lambda} - \lambda^i L_m^i) +$$

$$- \sum_{i=1}^{n}(\pi_r' + h_m)B^i(S_m^i, Q_m, L_m^i) +$$

$$- \sum_{i=1}^{n}\pi_r\lambda^i P^i(S_m^i, Q_m, L_m^i).\Omega_r^i(Q^i,$$

Note again that the supply chain costs are minimized or the channel is coordinated, only if the retailer charges the same backorder penalties that she observes, i.e., $\pi_m = \pi_r$ and $\pi'_m = \pi'_r$. Because, only in this case, the manufacturer (who makes the decision on Q and $\mathbf{S}$) and the supply chain have the same cost function (i.e., objective function).

For the retailer and the manufacturer to participate in the contract, both have to be better off with the contract. Thus, the following conditions should be satisfied.

$$\Omega_r(\mathbf{Q_r}, \mathbf{r_r}) \geq \Omega_r^c(Q_m, \mathbf{S_m}) \tag{3.58}$$

$$\Omega_m(\mathbf{Q_r}, \mathbf{r_r}) \geq \Omega_m^c(Q_m, \mathbf{S_m}). \tag{3.59}$$

These two conditions enforce upper and lower bound constraints on A. These bounds are:

$$\begin{aligned} A \geq & K\frac{\lambda}{Q_m} + \sum_{i=1}^{n}(h_m)(S_m^i - \frac{\lambda^i(Q_m-1)}{2\lambda} - \lambda^i L_m^i) \\ & + \sum_{i=1}^{n}(\pi'_m + h_m)B^i(S_m^i, Q_m, L_m^i) \\ & + \sum_{i=1}^{n}\pi_m\lambda^i P^i(S_m^i, Q_m, L_m^i) \end{aligned} \tag{3.60}$$

$$\begin{aligned} A \leq & \sum_{i=1}^{n}\left[K\frac{\lambda^i}{Q_r^i} + h_r\chi(Q_r^i, r_r^i, L_r^i) + \pi_r E(Q_r^i, r_r^i, L_r^i) + \pi'_r B(Q_r^i, r_r^i, L_r^i)\right] \\ & - \sum_{i=1}^{n}(\pi_r - \pi_m)\lambda^i P^i(S_m^i, Q_m, L_m^i) - \sum_{i=1}^{n}(\pi'_r - \pi'_m)B^i(S_m^i, Q_m, L_m^i). \end{aligned} \tag{3.61}$$

Note once again that a feasible A can be found, only if the supply chain cost savings are non-negative. The exact value of A specifies how the savings through the contract are allocated to the parties in the supply chain.

Chapter 4

Contracts Without Setup Costs

In this chapter, we construct and examine various contracts using the base stock model we introduced in Section **??**. We build four base cases to differentiate situations where different types of backorders (Type I or Type II) and different backorder costs (high or low) are incurred. In Table 4.1, base case parameters and in Table 4.2, the optimal solutions of base cases are given. Exact cost expressions are calculated using a program coded in C++ and Matlab and optimal solutions are found through enumeration. We take $\lambda = 5$ and $A = 0$ for all cases that we examine.

Table 4.1: Physical Improvement Base Case Parameters

Base Case	L_r	h_r	π_r	π'_r	K
1	2	6	100	0	0
2	2	6	0	100	0
3	2	6	50	0	0
4	2	6	0	50	0

Table 4.2: Physical Improvement Base Case Optimal S_r and Cost Components

		Cost Components				
Base Case	S_r	Ordering	Holding	BO_1	BO_2	Total
1	14	0	25.122	8.346	0	33.468
2	15	0	30.621	0	10.348	40.969
3	12	0	15.186	10.422	0	25.608
4	14	0	25.122	0	9.347	34.469

In Section 4.1, we analyze savings achieved through physical improvement. As we mentioned before, physical improvement consists of holding cost reduction and leadtime improvement. We analyze the savings achieved through leadtime reduction in Section 4.1.1, and through holding cost reduction, in Section 4.1.2. In Section 4.2, we examine the impact of the retailer charging different backorder costs on supply chain costs when physical improvements are provided. Physical improvements are obtained through leadtime reduction in Section 4.2.1, and through leadtime reduction in Section 4.2.2.

4.1 Physical Improvement Under Centralized Control

In this section we analyze physical improvements achieved through centralized control. Base cases exhibit before contract situations. After the manufacturer assumes control, the system is improved through either leadtime reduction or holding cost reduction. More savings would be achieved, if leadtime and holding cost were reduced at the same time, but in that case the marginal effects of those would not be captured. So each table is constructed by varying a single parameter. The cost structure of a single the retailer under base stock policy and cost expressions of parties after contract is given in Chapter 3. For the sake of simplicity, in this section we consider there is a single the retailer installation and its control is assumed by the manufacturer after the contract. Under centralized control, the backorder cost parameters that the manufacturer observes are exactly equal to the backorder cost parameters that the retailer sees. In other words, the retailer truly reflects its own backorder costs under centralized control and the retailer and the manufacturer act as a single entity. The channel costs are simply equal to the manufacturer's cost.

4.1.1 Leadtime Reduction

Table 4.3: Base Case 1 Percentage Savings - Leadtime Reduction

Base Case 1		Cost Components						
L_m	S_m	Ordering	Holding	BO_1	BO_2	Total	% Savings	Abs. Diff.
0.5	5	0	15.372	4.202	0	19.574	41.515	13.894
0.75	6	0	14.379	8.628	0	23.007	31.256	10.461
1	8	0	18.733	6.809	0	25.542	23.683	7.926
1.25	9	0	17.717	10.221	0	27.938	16.524	5.530
1.5	11	0	21.969	7.924	0	29.893	10.680	3.574
1.75	12	0	20.916	10.680	0	31.596	5.594	1.872
2	14	0	25.122	8.346	0	33.468	0	0

In Table 4.3, we consider base case 1. In this base case a high per occasion backorder cost (Type I) is incurred. At each step L_m is reduced 0.25 units. It is observed that, as leadtime gets smaller, the base stock level and the total cost decreases. However note that not all cost components decrease as the leadtime gets smaller. As L_m is decreased from 2 to 1.25, BO_1 increases from 8.346 to 10.221. But this increase is compensated by a larger decrease in holding cost where holding cost decrease from 25.122 to 17.717, so positive savings are achieved. A similar behavior can be observed in holding costs. As L_m is decreased from 1.25 to 1, holding cost increases from 17.717 to 18.733. This is due to discontinuous structure of the cost function. At discontinuity points, sudden shifts in backorder cost terms and holding cost terms are observed. This type of behavior is also in other cases later in this chapter. At most 41.515% savings are achieved when L_m is decreased to 0.5. The improvements achieved by reducing leadtime can also be observed through examining absolute differences in costs and those differences are given in Table 4.3. The absolute differences show a similar pattern to percentage savings.

Before going further, using the data we presented for base case 1, we calculate the bounds for the annual fee, A. Total cost presented in Table 4.3, provides a lower bound for the annual payment since for any fee less than relevant total cost, manufacturer has a positive cost, which is greater than her initial cost, 0. This implies that, any payment less than the lower bound is not profitable for the manufacturer. Similarly, for any fee that is greater than the initial cost of the retailer, 33.468, the partnership is not profitable for the retailer. For participation of both, the annual payment shall be between, the manufacturer cost and the initial retailer cost. The bounds for this base case is presented in Table 4.4.

Table 4.4: Base Case 1 Annual Payment Bounds - Leadtime Reduction

L_m	Lower B.	Upper B.
0.5	19.574	33.468
0.75	23.007	33.468
1	25.542	33.468
1.25	27.938	33.468
1.5	29.893	33.468
1.75	31.596	33.468
2	33.468	33.468

As it can be seen in Table 4.4, as leadtime is improved, the lower bound for the range decreases which creates a larger range for annual payment which in turn creates an increased opportunity for a contract. This result can be repeated for all numerical data that we present. We should note here that the annual payment alone may not be enough to determine whether the retailer or the manufacturer will participate in the contract. As we mentioned before, both retailer and the manufacturer may have additional benefits such as the strengthened market share for the manufacturer and the ability to divert the focus to its own operations for the retailer. Thus, the manufacturer or the retailer may still want to participate even though they may be increasing their operational costs.

Table 4.5: Base Case 2 Percentage Savings - Leadtime Reduction

Base Case 2		Cost Components					
L_m	S_m	Ordering	Holding	BO_1	BO_2	Total	% Savings
0.5	5	0	15.372	0	6.195	21.567	47.358
0.75	7	0	19.861	0	6.021	25.882	36.825
1	9	0	24.324	0	5.402	29.726	27.442
1.25	10	0	23.103	0	10.056	33.159	19.062
1.5	12	0	27.494	0	8.232	35.726	12.797
1.75	14	0	31.903	0	6.718	38.621	5.731
2	15	0	30.621	0	10.348	40.969	0

In Table 4.5, we consider base case 2, where backorder is incurred per item per time basis (Type II) rather than per occasion basis. At each step L_m is reduced 0.25 units. Note again that base stock levels and total costs get smaller as leadtime is reduced. However inventory holding and backorder costs are not individually monotonically decreasing. For example, as L_m is decreased from 1.75 to 1, BO_2 increases from 6.718 to 10.056. But this increase is compensated by a larger decrease in holding cost where holding cost decrease from 31.903 to 24.324, so positive savings are achieved. A similar behavior can be observed in holding costs. As L_m is decreased from 1.25 to 1, holding cost increases from 23.103 to 24.324. At most 47.358% savings are achieved when L_m is decreased to 0.5, which is even more greater than base case 1.

Base case 1 and 2 demonstrate situations where backorder costs are “high”. Now we examine base case 3 and 4 which demonstrate situations where backorder costs are “low”.

Table 4.6: Base Case 3 Percentage Savings - Leadtime Reduction

Base Case 3		Cost Components					
L_m	S_m	Ordering	Holding	BO_1	BO_2	Total	% Savings
0.5	4	0	10.025	5.441	0	15.466	39.605
0.75	5	0	9.442	8.856	0	18.298	28.546
1	7	0	13.533	6.669	0	20.201	21.112
1.25	8	0	12.794	8.981	0	21.775	14.965
1.5	9	0	12.138	11.180	0	23.317	8.945
1.75	11	0	15.956	8.672	0	24.628	3.826
2	12	0	15.186	10.422	0	25.608	0

In Table 4.6, we consider base case 3, where backorder cost incurred per occasion basis as in base case 1. At each step L_m is reduced 0.25 units. Results are similar to those in base case 1 are observed. The percentage savings in this case are less than the percentage savings in base case 1 for some lead time values and more than the percentage savings in base case 1 for some other lead time values.

Table 4.7: Base Case 4 Percentage Savings, Leadtime Reduction

Base Case 4		Cost Components					
L_m	S_m	Ordering	Holding	BO_1	BO_2	Total	% Savings
0.5	5	0	15.372	0	3.098	18.469	46.417
0.75	6	0	14.379	0	7.325	21.703	37.034
1	8	0	18.733	0	6.106	24.838	27.940
1.25	9	0	17.717	0	10.138	27.855	19.189
1.5	11	0	21.969	0	8.078	30.047	12.827
1.75	12	0	20.916	0	11.797	32.712	5.096
2	14	0	25.122	0	9.347	34.469	0

In Table 4.7, we consider base case 4, where backorder is incurred per unit per time basis as base case 2. At each step L_m is reduced 0.25 units. The results are similar to those in base case 2.

Note that leadtime reduction brings more percentage savings in cases where Type II backorder is incurred (base case 2 and 4) than cases where Type I backorder is incurred (base case 1 and 3). This is due to the fact that charging a fixed penalty per unit per time is more prohibitive than charging the same penalty per occasion. Hence lead time reduction is more effective and savings are more for the case of Type II backorder costs. We see that the difference between percentage savings decline (in percentage) as the leadtime reductions gets larger.

4.1.2 Holding Cost Reduction

Table 4.8: Base Case 1 Percentage Savings - Holding Cost Reduction

Base Case 1		Cost Components					
h_m	S_m	Ordering	Holding	BO_1	BO_2	Total	% Savings
3	15	0	15.310	4.874	0	20.184	39.690
3.25	15	0	16.586	4.874	0	21.460	35.877
3.5	15	0	17.862	4.874	0	22.736	32.065
3.75	15	0	19.138	4.874	0	24.012	28.253
4	14	0	16.748	8.346	0	25.094	25.021
4.25	14	0	17.794	8.346	0	26.140	21.893
4.5	14	0	18.841	8.346	0	27.187	18.766
4.75	14	0	19.888	8.346	0	28.234	15.638
5	14	0	20.935	8.346	0	29.281	12.510
5.25	14	0	21.981	8.346	0	30.327	9.383
5.5	14	0	23.028	8.346	0	31.374	6.255
5.75	14	0	24.075	8.346	0	32.421	3.128
6	14	0	25.122	8.346	0	33.468	0

In Table 4.8, we consider base case 1. In this base case a high per occasion backorder cost (Type I) is incurred. At each step h_m is reduced 0.25 units. It is observed that optimal base stock level chosen by the manufacturer, S_m increases since holding inventory becomes less costly. S_m increases from 14 to 15, meanwhile holding cost decreases from 25.122 to 15.310 as h_m is reduced to 3 from 6. As expected total cost decreases in this direction. The same tradeoff between holding costs and backorder costs is observed here as it is observed in leadtime reduction cases. As h_m is reduced to 3.75 from 4, inventory cost increases to 19.138 from 16.748 meanwhile BO_1 decreases to 4.874 from 8.346, which compensates the increase in holding costs and positive savings are achieved. Unlike leadtime reduction case the backorder costs decrease in monotonic manner as holding cost decreases. At most 39.690% savings are achieved when h_m is decreased to 3.

Similar to what we did in Section 4.1.1, we calculate the bounds for the annual fee, A, for base case 1. Total cost presented in Table 4.9, provides a lower bound for the annual payment since for any fee less than relevant total cost, manufacturer has a positive cost, which is greater than her initial cost, 0. This implies that, any payment less than the lower bound is not profitable for the manufacturer. Similarly, for any fee that is greater than the initial cost of the retailer, 33.468, the partnership is not profitable for the retailer. For participation of both, the annual payment shall be between, the manufacturer cost and the initial retailer cost. The bounds for this base case is presented in Table 4.9.

Table 4.9: Base Case 1 Annual Payment Bounds - Holding Cost Reduction

h_m	Lower B.	Upper B.
3	20.184	33.468
3.25	21.460	33.468
3.5	22.736	33.468
3.75	24.012	33.468
4	25.094	33.468
4.25	26.140	33.468
4.5	27.187	33.468
4.75	28.234	33.468
5	29.281	33.468
5.25	30.327	33.468
5.5	31.374	33.468
5.75	32.421	33.468
6	33.468	33.468

As it can be seen in Table 4.9, as holding cost is improved, the lower bound for the range decreases which creates a larger range for annual payment which in turn creates an increased opportunity for a contract.

Table 4.10: Base Case 2 Percentage Savings - Holding Cost Reduction

Base Case 2		Cost Components					
h_m	S_m	Ordering	Holding	BO_1	BO_2	Total	% Savings
3	16	0	18.164	0	5.474	23.638	42.302
3.25	16	0	19.678	0	5.474	25.152	38.607
3.5	16	0	21.192	0	5.474	26.666	34.913
3.75	16	0	22.705	0	5.474	28.179	31.218
4	16	0	24.219	0	5.474	29.693	27.523
4.25	16	0	25.733	0	5.474	31.207	23.828
4.5	16	0	27.246	0	5.474	32.720	20.134
4.75	16	0	28.760	0	5.474	34.234	16.439
5	16	0	30.274	0	5.474	35.748	12.744
5.25	15	0	26.793	0	10.348	37.141	9.343
5.5	15	0	28.069	0	10.348	38.417	6.228
5.75	15	0	29.345	0	10.348	39.693	3.114
6	15	0	30.621	0	10.348	40.969	0

In Table 4.10, we consider base case 2. At each step h_m is reduced 0.25 units. In this base case a high per unit per time backorder cost (Type II) is incurred. As holding cost is reduced, S_m increases since now more inventory could be kept with less lower cost. Consequently backorder costs decline. As base stock level shifts from 15 to 16, holding cost increases slightly but this is compensated by a sharp decrease in backorder cost likewise in base case 1. At most 42.302% savings are achieved when h_m is decreased to 3. Note that more percentage savings is achieved than base case 1 due to difference in types of backorders. Similar to our findings in Section 4.1.1, this time reduction in holding cost brings more savings when backorders are incurred on per unit per time basis.

Table 4.11: Base Case 3 Percentage Savings - Holding Cost Reduction

Base Case 3		Cost Components					
h_m	S_m	Ordering	Holding	BO_1	BO_2	Total	% Savings
3	14	0	12.561	4.173	0	16.734	34.653
3.25	13	0	10.798	6.777	0	17.575	31.368
3.5	13	0	11.629	6.777	0	18.406	28.124
3.75	13	0	12.459	6.777	0	19.236	24.880
4	13	0	13.290	6.777	0	20.067	21.637
4.25	13	0	14.120	6.777	0	20.897	18.393
4.5	13	0	14.951	6.777	0	21.728	15.149
4.75	12	0	12.022	10.422	0	22.444	12.354
5	12	0	12.655	10.422	0	23.077	9.884
5.25	12	0	13.287	10.422	0	23.709	7.413
5.5	12	0	13.920	10.422	0	24.342	4.942
5.75	12	0	14.553	10.422	0	24.975	2.471
6	12	0	15.186	10.422	0	25.608	0

In Table 4.11, we consider base case 3. In this base case a low per occasion backorder cost (Type I) is incurred. As holding cost is reduced, S_m increases since now more inventory could be kept with less price. Consequently backorder costs reduce since now there are less stockout situations. As base stock level shifts values (such as 12 to 13, 13 to 14), holding cost increases slightly but this is compensated by a sharp decrease in backorder cost as observed in base case 1 and 2. At most 34.653% savings are achieved when h_m is decreased to 3.

Table 4.12: Base Case 4 Percentage Savings, Holding Cost Reduction

Base Case 4		Cost Components					
h_m	S_m	Ordering	Holding	BO_1	BO_2	Total	% Savings
3	15	0	15.310	0	5.174	20.484	40.571
3.25	15	0	16.586	0	5.174	21.760	36.869
3.5	15	0	17.862	0	5.174	23.036	33.168
3.75	15	0	19.138	0	5.174	24.312	29.466
4	15	0	20.414	0	5.174	25.588	25.765
4.25	15	0	21.690	0	5.174	26.864	22.063
4.5	15	0	22.966	0	5.174	28.140	18.362
4.75	14	0	19.888	0	9.347	29.235	15.184
5	14	0	20.935	0	9.347	30.282	12.147
5.25	14	0	21.981	0	9.347	31.328	9.110
5.5	14	0	23.028	0	9.347	32.375	6.074
5.75	14	0	24.075	0	9.347	33.422	3.037
6	14	0	25.122	0	9.347	34.469	0

In Table 4.12, we consider base case 4. In this base case a low per unit per time backorder cost (Type II) is incurred. At each step h_m is reduced 0.25 units. The results are similar to those in Table 4.10 for base case 2. At most 40.571% savings are achieved when h_m is decreased to 3. Again from comparison of base case 3 and 4 under holding cost reduction, it can be deduced that holding cost improvement brings more percentage savings when Type II backorder costs are incurred.

Until now, we have shown that considerable savings are achievable through physical improvement. Both leadtime reduction and holding cost reduction can be used to achieve savings around 40% when L_m is reduced to 0.5 from 4 or h_m is reduced to 3 from 6. Another result that we identified is when backorders are "high", physical improvement brings more percentage savings. We have also shown that physical improvement works better when backorder costs are incurred on per unit per time basis rather than per occasion basis. We identified that the discontinuity shifts in S_m, causes sudden increases in holding costs and decreases backorder costs. In our data sets, holding cost reduction brought more savings than leadtime improvement.

As holding cost is reduced, inventory levels increase while cost of holding such large inventories decrease which in turn reduces the backorders due to decreased number of stockouts. But in leadtime reduction case, as base stock levels decrease due to shorter leadtime, the backorders may increase and hamper the savings. In the next section we investigate the situation where the retailer manipulates backorder costs to achieve savings and the "limits" to this manipulation.

4.2 Decentralized Control

In this section, we study the impact of the retailer charging a different backorder penalty than what she observes on coordination of the channel. As we defined in Chapter 3, the retailer pays her customers π_r and π'_r but in the contract she may charge the manufacturer backorder costs which are different (i.e. $\pi_m \neq \pi_r$ and $\pi'_m \neq \pi'_r$). This manipulation can be done in various ways. First the backorder cost may be changed without changing the type of the backorder cost. For example, if the retailer is charged per occasion basis by customer, the retailer may charge the manufacturer again on per occasion basis but with a different cost. Second, the retailer may charge a different type of backorder cost (possibly with a different amount than what she faces) to the manufacturer, such as charging Type II backorder cost while observing Type I backorder cost. Again we define 4 base cases to demonstrate behavior of cost functions of the retailer, the manufacturer and supply chain. The backorder cost ranges that are incurred to the manufacturer are given in the Table 4.13. The optimal solution of base cases before contract are given in Table 4.14.

Table 4.13: Decentralized Channel - Base Case Parameters

Base Case	L_r	h_r	π_r	π'_r	K	π_m	π'_m	K
1	2	6	0	100	0	0	[50, 150]	0
2	2	6	0	50	0	0	[25, 75]	0
3	2	6	50	0	0	[25, 75]	0	0
4	2	6	0	50	0	[50, 150]	0	0

Table 4.14: Decentralized Channel - Base Case Optimal S_r and Cost Components

		Cost Components				
Base Case	S_r	Ordering	Holding	BO_1	BO_2	Total
1	15	0	30.621	0	10.348	40.969
2	14	0	25.122	0	9.347	34.469
3	12	0	15.186	10.422	0	25.608
4	14	0	25.122	0	9.347	34.469

In Section 4.2.1, we investigate the situation when the physical improvement is achieved through leadtime reduction. In Section 4.2.2, we repeat the same analysis in a setting where physical improvement is achieved through holding cost reduction.

4.2.1 Decentralized Control with Leadtime Reduction

Table 4.15: Base Case 1 Percentage Savings, $L_m = 1.5$, π'_m changes

	Total Costs			Savings
π'_m	Retailer	Manufacturer	Supply Chain	%
50	8.078	30.047	38.125	6.9
60	6.462	31.663	38.125	6.9
70	2.470	33.256	35.726	12.8
80	1.646	34.080	35.726	12.8
90	0.823	34.903	35.726	12.8
100	0	35.726	35.726	12.8
110	-0.823	36.549	35.726	12.8
120	-1.646	37.372	35.726	12.8
130	-2.470	38.196	35.726	12.8
140	-1.586	38.789	37.203	9.2
150	-1.983	39.185	37.203	9.2

In Table 4.15, we consider base case 1. When the manufacturer assumes the control, leadtime is reduced to 1.5. Type II backorder cost charged by the retailer to the manufacturer, is iterated between 50 and 150 with increments of 10. The costs of the manufacturer, the retailer and channel are also given in the table 4.15. Note that these costs exclude the annual fee that is paid by the retailer to the manufacturer. Even if the retailer charges the manufacturer $\pi'_m = 50$, which is much less than what she observes, positive channel savings are possible (6.9%). This indicates the considerable effect of leadtime reduction. When $70 \leq \pi'_m \leq 130$, minimum channel cost, 35.726, and maximum percentage savings in the channel, 12.8%, are achieved. For $\pi'_m > 100$, the retailer "earns" money from backorders, which explains the negative values that is seen in the retailer's costs. Note that for those values, the manufacturer's cost is greater than the channel cost. As for backorder costs that are greater 130 and smaller than 70, channel savings diminish to 9.2 and 6.9 respectively from 12.8. An important observation is that channel coordination is achieved in an interval around $\pi'_m = 100$. We obtained an interval for π_m where all values in that interval, coordinate the channel. Note that the channel is coordinated for a range of π_m values and $\pi'_m = 100$, which is also equal to π'_r, is in that interval too.

Table 4.16: Base Case 2 Percentage Savings, $L_m = 1.5$, π'_m changes

	Total Costs			Savings
π'_m	Retailer	Manufacturer	Supply Chain	%
25	7.483	24.279	31.762	7.9
30	5.986	25.776	31.762	7.9
35	4.490	27.272	31.762	7.9
40	1.616	28.432	30.047	12.8
45	0.808	29.240	30.047	12.8
50	0	30.047	30.047	12.8
55	-0.808	30.855	30.047	12.8
60	-1.616	31.663	30.047	12.8
65	-2.423	32.471	30.047	12.8
70	-1.646	33.256	31.610	8.3
75	-2.058	33.668	31.610	8.3

In Table 4.16, we consider base case 2. This case is very similar to base case 1, only difference is this time the retailer faces a lower Type II backorder cost.

Again leadtime is reduced to 1.5 under the manufacturer control. Type II backorder cost charged by the retailer to the manufacturer, is iterated between 25 and 75 by increments of 5. The costs of the manufacturer, the retailer and channel are given in the table 4.16. As it is shown in the table, even if the retailer charges the manufacturer $\pi'_m = 25$, which is much less than what she observes, positive channel savings are possible (7.9%). When $40 \leq \pi'_m \leq 65$, minimum channel cost, 30.047, and maximum percentage savings in the channel, 12.8%, are achieved. For $\pi'_m > 65$, the retailer "earns" money from backorders, which explains the negative values that is seen in the retailer's costs. Note that for those values, the manufacturer's cost is greater than the channel cost. As for backorder costs that are greater than 65 and smaller than 40, channel savings diminish to 8.3 and 7.9 respectively from 12.8. Again channel coordination is achieved in an interval around $\pi'_m = 50$ which supports our observation in base case 1. Note that similar percentage savings are achieved as base case 1.

Table 4.17: Base Case 3 Percentage Savings, $L_m = 1.5$, π_m changes

	Total Costs			Savings
π_m	Retailer	Manufacturer	Supply Chain	%
25	8.451	16.616	25.067	2.1
30	6.761	18.307	25.067	2.1
35	3.354	19.963	23.317	8.9
40	2.236	21.081	23.317	8.9
45	1.118	22.199	23.317	8.9
50	0	23.317	23.317	8.9
55	-0.689	24.373	23.684	7.5
60	-1.378	25.062	23.684	7.5
65	-2.066	25.75	23.684	7.5
70	-2.755	26.439	23.684	7.5
75	-3.444	27.128	23.684	7.5

In Table 4.17, we consider base case 3. In this case the retailer faces a lower Type I backorder cost. Leadtime is reduced to 1.5 under the manufacturer control. Type I backorder cost charged by the retailer to the manufacturer, is iterated between

25 and 75 by increments of 5. The costs of the manufacturer, the retailer and channel are given in the Table 4.17. The results are similar to those for base case 1. The maximum savings are possible when the retailer charges the same backorder penalties that she observes.

Table 4.18: Base Case 4 Percentage Savings, $L_m = 1.5$, π_m changes

	Total Costs			Savings
π_m	Retailer	Manufacturer	Supply Chain	%
25	34.597	16.616	51.213	-48.6
30	32.907	18.307	51.213	-48.6
35	18.320	19.963	38.284	-11.1
40	17.202	21.081	38.284	-11.1
45	16.084	22.199	38.284	-11.1
50	14.967	23.317	38.284	-11.1
55	7.389	24.373	31.762	7.9
60	6.700	25.062	31.762	7.9
65	6.012	25.75	31.762	7.9
70	5.323	26.439	31.762	7.9
75	4.634	27.128	31.762	7.9
80	3.945	27.817	31.762	7.9
85	3.256	28.506	31.762	7.9
90	0.946	29.101	30.047	12.8
95	0.550	29.497	30.047	12.8
100	0.154	29.893	30.047	12.8
105	-0.242	30.290	30.047	12.8
110	-0.638	30.686	30.047	12.8
115	-1.035	31.082	30.047	12.8
120	-1.431	31.478	30.047	12.8
125	-1.827	31.874	30.047	12.8
130	-2.223	32.271	30.047	12.8
135	-2.619	32.667	30.047	12.8
140	-3.016	33.063	30.047	12.8
145	-3.412	33.459	30.047	12.8
150	-3.808	33.855	30.047	12.8

In Table 4.18, we consider base case 4. In this base case, the retailer faces Type II backorder and incurs Type I backorder to the manufacturer. This scenario demonstrates a case where the retailer maybe facing backorder costs on a per unit per time basis, but measurement of this fact is not possible or practical under a contract. Therefore, the manufacturer is only charged by each occurrence of a backorder. Leadtime is reduced to 1.5 under the manufacturer control. Type I backorder cost charged by the retailer to the manufacturer, is iterated between 25 and 150 by increments of 5. The costs of the manufacturer, the retailer and channel are given in the Table 4.18. Unlike the previous base cases, if the retailer charges too low, the channel may be worse off. For example if $\pi_m = 25$, channel costs increase by 48.6%. This happens even though 50% reduction in leadtime is obtained under manufacturer control. When a stockout is realized, the retailer pays a greater cost every item that is included in that backorder and this fact is unobserved to the manufacturer. For this reason the retailer must find an appropriate backorder penalty to charge the manufacturer and force her to keep more stock. The results given in Table 4.18 shows that the maximum channel savings, 12.8% and minimum channel cost, 30.047, are achieved when $90 \leq \pi_m \leq 150$.

4.2.2 Decentralized Control with Holding Cost Reduction

Table 4.19: Base Case 1 Percentage Savings, $h_m = 4$, π'_m changes

	Total Costs			Savings
π'_m	Retailer	Manufacturer	Supply Chain	%
50	5.174	25.588	30.762	24.9
60	4.139	26.623	30.762	24.9
70	3.104	27.658	30.762	24.9
80	1.095	28.598	29.693	27.5
90	0.547	29.146	29.693	27.5
100	0	29.693	29.693	27.5
110	-0.547	30.240	29.693	27.5
120	-1.095	30.788	29.693	27.5
130	-1.642	31.335	29.693	27.5
140	-2.190	31.883	29.693	27.5
150	-1.385	32.266	30.881	24.6

In Table 4.19, we consider base case 1. When the manufacturer assumes the control, holding cost is reduced to 4. Type II backorder cost charged by the retailer to the manufacturer, is iterated between 50 and 150 by increments of 10. The costs of the manufacturer, the retailer and channel are also given in the table 4.19. As it is shown in the table, even if the retailer charges the manufacturer $\pi'_m = 50$, which is much less than what she observes, positive channel savings are possible (24.9%). This indicates the even more greater than the effect of holding cost reduction. When $80 \leq \pi'_m \leq 140$, minimum channel cost, 29.693, and maximum percentage savings in the channel, 27.5%, are achieved. For $\pi'_m > 100$, the retailer "earns" money from backorders, which explains the negative values that is seen in the retailer's costs. Note that for those values, the manufacturer's cost is greater than the channel cost. An important observation is that channel coordination is achieved in an interval around $\pi'_m = 100$. We obtained an interval for π_m where all values in that interval, coordinate the channel. As for backorder costs that are greater 140 and smaller than 80, channel savings diminish to 24.6% and 24.9% respectively from 27.5%.

Table 4.20: Base Case 2 Percentage Savings, $h_m = 4$, π'_m changes

	Total Costs			Savings
π'_m	Retailer	Manufacturer	Supply Chain	%
25	8.062	21.352	29.413	14.7
30	3.739	22.356	26.095	24.3
35	2.804	23.291	26.095	24.3
40	1.869	24.225	26.095	24.3
45	0.517	25.071	25.588	25.8
50	0	25.588	25.588	25.8
55	-0.517	26.105	25.588	25.8
60	-1.035	26.623	25.588	25.8
65	-1.552	27.140	25.588	25.8
70	-2.070	27.658	25.588	25.8
75	-2.587	28.175	25.588	25.8

In Table 4.20, we consider base case 2. This case is very similar to base case 1, only difference is this time the retailer faces a lower Type II backorder cost. Again holding cost is reduced to 4 under the manufacturer control. Type II backorder cost charged by the retailer to the manufacturer, is iterated between 25 and 75 by increments of 5. The results are similar to those obtained in Table 4.19, except this time, the channel is coordinated in a wider range.

Table 4.21: Base Case 3 Percentage Savings, $h_m = 4$, π_m changes

	Total Costs			Savings
π_m	Retailer	Manufacturer	Supply Chain	%
25	7.581	14.917	22.498	12.1
30	4.169	16.377	20.546	19.8
35	3.127	17.419	20.546	19.8
40	2.084	18.461	20.546	19.8
45	0.678	19.389	20.067	21.6
50	0	20.067	20.067	21.6
55	-0.678	20.745	20.067	21.6
60	-1.355	21.422	20.067	21.6
65	-2.033	22.100	20.067	21.6
70	-1.669	22.590	20.921	18.3
75	-2.087	23.007	20.921	18.3

In Table 4.21, we consider base case 3. In this case the retailer faces a lower Type I backorder cost. Holding cost is reduced to 4 under the manufacturer control. Type I backorder cost charged by the retailer to the manufacturer, is iterated between 25 and 75 by increments of 5. The costs of the manufacturer, the retailer and channel are given in the Table 4.21. As it is shown in the table, even if the retailer charges the manufacturer $\pi_m = 25$, which is much less than what she observes, positive channel savings are possible (12.1%). This again indicates the effect of leadtime reduction. When $45 \leq \pi_m \leq 65$, minimum channel cost, 20.067, and maximum percentage savings in the channel, 21.6%, are achieved. For $\pi_m > 50$, the retailer "earns" money from each stockout situation, which explains the negative values that is seen in the retailer's costs. In this case lower overall savings are achieved due to change in the backorder cost type. Channel coordination is achieved in an interval around the original per occasion backorder cost, $\pi_r = 50$, which supports that without the retailer manipulating backorder costs, the channel has the most savings.

Table 4.22: Base Case 4 Percentage Savings, $h_m = 4$, π_m changes

	Total Costs			Savings
π_m	Retailer	Manufacturer	Supply Chain	%
25	34.127	14.917	49.044	-42.3
30	20.293	16.377	36.670	-6.4
35	19.251	17.419	36.670	-6.4
40	18.208	18.461	36.670	-6.4
45	10.024	19.389	29.413	14.7
50	9.347	20.067	29.413	14.7
55	8.669	20.745	29.413	14.7
60	7.991	21.422	29.413	14.7
65	7.313	22.100	29.413	14.7
70	3.505	22.590	26.095	24.3
75	3.088	23.007	26.095	24.3
80	2.670	23.425	26.095	24.3
85	2.253	23.842	26.095	24.3
90	1.836	24.259	26.095	24.3
95	1.418	24.676	26.095	24.3
100	1.001	25.094	26.095	24.3
105	0.584	25.511	26.095	24.3
110	-0.187	25.775	25.588	25.8
115	-0.431	26.019	25.588	25.8
120	-0.675	26.263	25.588	25.8
125	-0.919	26.506	25.588	25.8
130	-1.162	26.75	25.588	25.8
135	-1.406	26.994	25.588	25.8
140	-1.650	27.238	25.588	25.8
145	-1.893	27.481	25.588	25.8
150	-2.137	27.725	25.588	25.8

In Table 4.22, we consider base case 4. In this base case, the retailer faces Type II backorder cost and incurs Type I backorder cost to the manufacturer. Holding cost is reduced to 4 under the manufacturer control. Type I backorder cost charged by the retailer to the manufacturer, is iterated between 25 and 150 by increments of 5. The costs of the manufacturer, the retailer and channel are given in the Table 4.22. Unlike the previous base cases, if the retailer charges too low, the channel loses money, even though the cost of owning the inventory is reduced considerably under manufacturer's control. For example if $\pi_m = 25$, channel savings increase by is 42.3%. In order to obtain savings for the channel, the retailer needs to find an appropriate Type I backorder penalty. The results given in Table 4.22 supports this identification since maximum channel savings, 25.8% and minimum channel cost, 25.588, is achieved when $110 \leq \pi_m \leq 150$.

Chapter 5

Contracts with Setup Costs

In this chapter, we conduct a numerical study for the case when there are positive setup costs for ordering. Before the contract, the retailer manages multiple installations independently using (r, Q) policy at each installation. After the contract, the manufacturer takes over the control, and manages multiple installations jointly using a $(Q, \mathbf{S})$ policy. The mathematical analysis of (r, Q) and $(Q, \mathbf{S})$ policies are given in Chapter 3. In Section 5.1, we study the impact of the joint replenishment alone on supply chain costs when the supply chain is under centralized control. In Section 5.2, in addition to the ability to jointly replenish multiple installations, the impact of further improvement through lead time reduction and inventory holding cost reduction is studied. In Section 5.3, we consider a decentralized control scenario and study the impact of the retailer charging backorder penalties different than she observers.

For simplicity, we assume that the retailer has two identical installations. Hence, the optimal policy parameters are also identical for these installations. The retailer or supply chain costs before the contract in all numerical examples in this chapter refer to the total cost in both installations (two times total cost of a single installation). Note that base stock level, S_m, stands for base stock levels at a single installation. In all numerical examples, we assume $A = 0$.

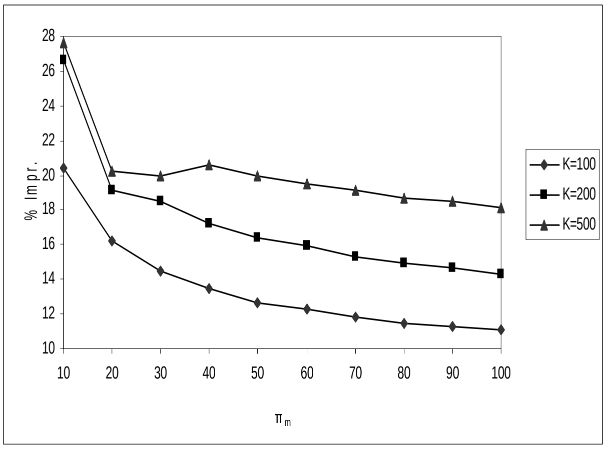
5.1 Effect of Pure JRP

In this section we assume the centralized control of the chain, thus the backorder penalties that are exactly equal to backorder penalties that the retailer observes. Also there are no physical improvements in the system. Hence, $\pi_m = \pi_r$, $\pi'_m = \pi'_r$, $h_m = h_r$ and $L_m = L_r$. We quantify the savings of the channel when manufacturer jointly manages inventories and uses $(Q, \mathbf{S})$ policy instead of (r, Q) policy after she assumes the control of the inventory. In each table we consider various factors such as holding cost, leadtime, Type I backorder cost or Type II backorder cost. We analyze each situation for $K = 100, 200, 500$ which stands for low, middle and high setup costs. In each iteration we calculate the optimal channel cost under (r, Q) policy and $(Q, \mathbf{S})$ policy. When we feed the parameters to $(Q, \mathbf{S})$ model, we directly obtain the channel cost. Under (r, Q) policy we simply calculate the total cost for a single retailer and multiply it by two to obtain channel cost since two installations are identical in all manners. Both costs and percentage difference of $(Q, \mathbf{S})$ cost from (r, Q) cost are given in tables.

Table 5.1: Pure JRP Savings - $\pi'_m = 0$, $h_m = 6$, $L_m = 2$

	K=100			K=200			K=500		
π_m	(r, Q)	$(Q, \mathbf{S})$	%	(r, Q)	$(Q, \mathbf{S})$	%	(r, Q)	$(Q, \mathbf{S})$	%
10	137.585	109.537	20.386	183.323	134.537	26.612	289.695	209.537	27.670
20	172.975	144.896	16.233	218.948	177.044	19.139	316.333	252.336	20.231
30	186.277	159.307	14.478	239.531	195.305	18.464	341.393	273.098	20.005
40	194.497	168.311	13.463	248.919	206.049	17.222	360.175	285.901	20.622
50	200.312	174.902	12.685	255.511	213.692	16.367	368.661	294.972	19.988
60	205.164	179.916	12.306	260.927	219.424	15.906	374.777	301.615	19.521
70	208.616	183.989	11.805	264.600	224.133	15.294	379.780	306.998	19.164
80	211.669	187.500	11.418	268.121	228.076	14.936	383.449	311.830	18.678
90	214.722	190.527	11.268	271.042	231.383	14.632	387.020	315.497	18.480
100	217.010	193.026	11.052	273.332	234.207	14.314	389.624	318.911	18.149

Figure 5.1: Pure JRP Savings - $\pi'_m = 0$, $h_m = 6$, $L_m = 2$

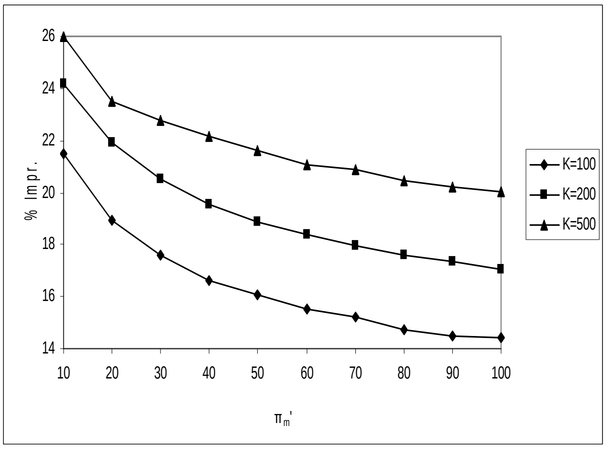


In Table 5.1, we fix the following parameters: $\pi'_m = 0$, $h_m = 6$, $L_m = 2$ and $K = 100, 200, 500$. We iterate π_m from 10 to 100 by 10 units at each step. Figure 5.1 demonstrates the change in percentage savings. In all cases $(Q, \mathbf{S})$ policy provided a smaller channel costs hence all percentage savings are positive and considerable. Note that largest deviation between (r, Q) and $(Q, \mathbf{S})$ policies is observed when $K = 500$ which indicates that as setup cost increases, joint replenishment brings more savings through joint ordering. Also note that the deviation diminishes as π_m increases. This is due to fact that orders can be triggered only jointly, when a total of Q demand occurs in $(Q, \mathbf{S})$ model, while the independent (r, Q) policy is able to trigger orders independently when there is a stockout. As π_m increases percentage savings decrease monotonically when $K = 100, 200$. However when $K = 500$, as π_m goes to 40 from 30, percentage savings increase to 20.622 from 20.005. For the remaining values, percentage savings continue to diminish monotonically. This phenomena results from the discrete nature of the problem. At that point, optimal $(Q, \mathbf{S})$ parameters change but (r, Q) parameters do not change. The change in $(Q, \mathbf{S})$ optimal parameters carries the system to a point where more percentage savings are achieved.

Table 5.2: Pure JRP Savings - $\pi_m = 0$, $h_m = 6$, $L_m = 2$

	K=100			K=200			K=500		
π'_m	(r,Q)	$(Q,\mathbf{S})$	%	(r,Q)	$(Q,\mathbf{S})$	%	(r,Q)	$(Q,\mathbf{S})$	%
10	131.754	103.411	21.512	179.999	136.479	24.178	287.250	212.635	25.976
20	151.695	122.975	18.933	204.134	159.351	21.938	311.519	238.350	23.488
30	162.652	134.021	17.602	216.350	171.971	20.513	327.118	252.644	22.767
40	169.992	141.736	16.622	224.489	180.566	19.566	337.169	262.413	22.172
50	175.574	147.385	16.055	230.595	187.037	18.889	344.153	269.806	21.603
60	180.021	152.108	15.505	235.480	192.236	18.364	349.556	275.842	21.088
70	183.792	155.862	15.197	239.354	196.368	17.959	354.300	280.377	20.865
80	186.572	159.052	14.750	242.562	199.894	17.591	357.747	284.637	20.436
90	189.352	161.870	14.514	245.739	203.094	17.354	361.024	288.142	20.187
100	192.124	164.460	14.399	248.099	205.862	17.024	364.103	291.074	20.057

Figure 5.2: Pure JRP Savings - $\pi_m = 0$, $h_m = 6$, $L_m = 2$

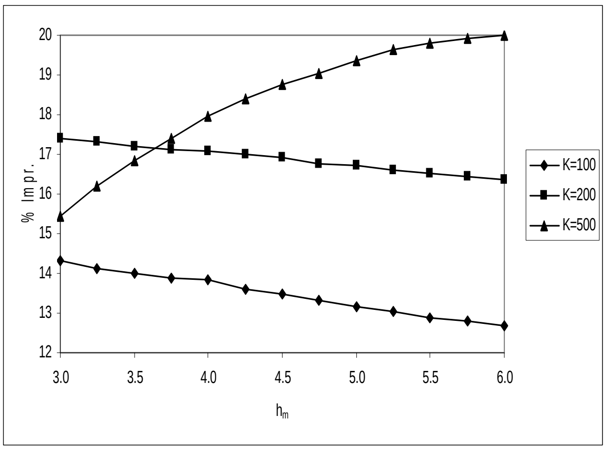


In Table 5.2, we fix the following parameters: $\pi_m = 0$, $h_m = 6$, $L_m = 2$ and $K = 100, 200, 500$. This time we iterate π'_m from 10 to 100 by 10 units at each step. Figure 5.2 demonstrates the change in percentage savings. In all cases $(Q, \mathbf{S})$ policy provides a smaller channel costs hence all percentage savings are positive. As the case of Type I backorder costs, the percentage savings decline as true backorder penalties are more positive. Also similar to the previous case, larger savings occur for larger setup costs.

Table 5.3: Pure JRP Savings - $\pi_m = 50$, $\pi'_m = 0$, $L_m = 2$

	K=100			K=200			K=500		
h	(r,Q)	$(Q,\mathbf{S})$	%	(r,Q)	$(Q,\mathbf{S})$	%	(r,Q)	$(Q,\mathbf{S})$	%
3	136.666	117.104	14.314	177.899	146.943	17.401	262.530	221.956	15.455
3.25	142.931	122.747	14.122	185.569	153.441	17.313	272.806	228.613	16.199
3.5	148.950	128.087	14.007	192.979	159.765	17.211	282.935	235.270	16.847
3.75	154.828	133.313	13.896	200.162	165.923	17.106	292.727	241.768	17.408
4	160.598	138.341	13.859	207.152	171.752	17.089	302.230	247.943	17.962
4.25	165.935	143.369	13.599	213.928	177.522	17.018	311.447	254.117	18.408
4.5	171.219	148.140	13.479	220.309	183.064	16.906	320.392	260.291	18.759
4.75	176.269	152.802	13.313	226.542	188.553	16.769	329.094	266.465	19.031
5	181.306	157.464	13.150	232.587	193.731	16.706	337.571	272.179	19.371
5.25	186.176	161.919	13.029	238.502	198.909	16.601	345.736	277.878	19.627
5.5	190.965	166.332	12.899	244.299	203.938	16.521	353.561	283.576	19.794
5.75	195.754	170.669	12.814	249.962	208.867	16.441	361.189	289.274	19.911
6	200.312	174.902	12.685	255.511	213.692	16.367	368.661	294.972	19.988

Figure 5.3: Pure JRP Savings - $\pi_m = 50$, $\pi'_m = 0$, $L_m = 2$



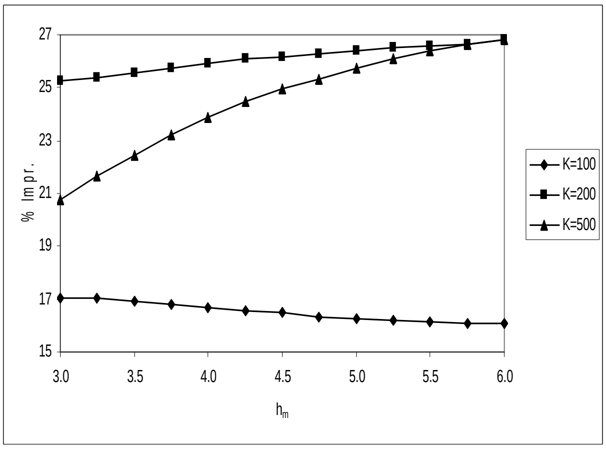
In Table 5.3, we fix the following parameters: $\pi_m = 50$, $\pi'_m = 0$, $L_m = 2$ and $K = 100, 200, 500$. This time we iterate h_m from 3 to 6 by 0.25 units at each step.

Figure 5.3 demonstrates the change in percentage savings. In all cases $(Q, \mathbf{S})$ policy provided a smaller channel costs hence all percentage savings are positive. However in this case an interesting observation is made. When $K = 100$, the deviation between (r, Q) and $(Q, \mathbf{S})$ decrease as holding cost increases. As cost of holding inventory becomes more and more expensive, base stock levels and order quantities decrease. Consequently backorder costs increase due to increased number of stockouts. Backorder cost increase more under $(Q, \mathbf{S})$ policy because of the "order delaying". Thus, the deviation of $(Q, \mathbf{S})$ policy from (r, Q) policy decreases. A similar situation is observed when $K = 200$ but this time a larger deviation is observed since $(Q, \mathbf{S})$ policy performance is enhanced under large setup cost. However when $K = 500$, as h_m advances from 3 to 6, the deviation increases. Under large setup costs, (r, Q) policy keeps larger inventories than $(Q, \mathbf{S})$ policy to prevent frequent ordering so holding cost under (r, Q) policy is considerably greater than holding cost under $(Q, \mathbf{S})$ policy. $(Q, \mathbf{S})$ policy also provides a lower setup cost since it exploits the advantages of joint replenishment. When $K = 500$ these two cost terms dominate the disadvantageous backorder cost of $(Q, \mathbf{S})$ policy related to due to the ability to only jointly trigger orders hence the difference between (r, Q) and $(Q, \mathbf{S})$ increases. Figure 5.3 presents this situation very clearly.

Table 5.4: Pure JRP Savings - $\pi_m = 0$, $\pi'_m = 50$, $L_m = 2$

	K=100			K=200			K=500		
h	(r,Q)	$(Q,\mathbf{S})$	%	(r,Q)	$(Q,\mathbf{S})$	%	(r,Q)	$(Q,\mathbf{S})$	%
3	124.049	102.931	17.024	177.899	132.984	25.247	262.530	208.037	20.757
3.25	129.318	107.275	17.046	185.569	138.434	25.400	272.806	213.735	21.653
3.5	134.348	111.595	16.936	192.979	143.664	25.555	282.935	219.434	22.444
3.75	139.006	115.687	16.775	200.162	148.626	25.747	292.727	224.773	23.214
4	143.572	119.664	16.652	207.152	153.470	25.914	302.230	230.005	23.898
4.25	147.914	123.410	16.567	213.928	158.109	26.092	311.447	235.236	24.470
4.5	152.213	127.128	16.480	220.309	162.706	26.146	320.392	240.468	24.946
4.75	156.311	130.845	16.292	226.542	166.978	26.293	329.094	245.699	25.341
5	160.409	134.312	16.270	232.587	171.222	26.384	337.571	250.702	25.734
5.25	164.288	137.679	16.197	238.502	175.322	26.490	345.736	255.478	26.106
5.5	168.142	141.046	16.115	244.299	179.320	26.598	353.561	260.254	26.391
5.75	171.962	144.262	16.109	249.962	183.319	26.661	361.189	265.030	26.623
6	175.574	147.385	16.055	255.511	187.037	26.799	368.661	269.806	26.815

Figure 5.4: Pure JRP Savings - $\pi_m = 0$, $\pi'_m = 50$, $L_m = 2$



In Table 5.4, we fix the following parameters: $\pi_m = 0$, $\pi'_m = 50$, $L_m = 2$ and $K = 100, 200, 500$. This time, we repeat the previous analysis but change the backorder type to Type II. Again we iterate h_m from 3 to 6 by 0.25 units at each step. Figure 5.4 demonstrates the change in percentage savings. In all cases $(Q, \mathbf{S})$ policy provided a smaller channel costs hence all percentage savings are positive. The deviation between (r, Q) and $(Q, \mathbf{S})$ is even more clear this time. A similar behavior to what is observed in Table 5.3 can be observed here. When $K = 100$, the deviation between (r, Q) and $(Q, \mathbf{S})$ decrease as holding cost increases. As cost of holding inventory becomes more and more expensive, base stock levels and order quantities decrease. Consequently backorder costs increase due to increased number of items in stockout position. Again backorder costs increase more under $(Q, \mathbf{S})$ policy because of the manufacturer's the ability to only jointly trigger orders. Thus, the deviation of $(Q, \mathbf{S})$ policy from (r, Q) policy decreases. In this case the difference between holding costs and setup costs of (r, Q) policy and $(Q, \mathbf{S})$ policy is also observed when $K = 200$. Again $(Q, \mathbf{S})$ policy performance is enhanced under large setup cost. When $h_m = 3$, there is a remarkable difference in deviations observed in $K = 200$ and $K = 500$ situations. When $K = 200$, a remarkably lower total cost is achieved even in lower holding costs is achieved since its performance is not hampered by increased number of items in stockout condition.

Figure 5.5: Pure JRP Savings - $\pi_m = 50$, $\pi'_m = 0$, h=6

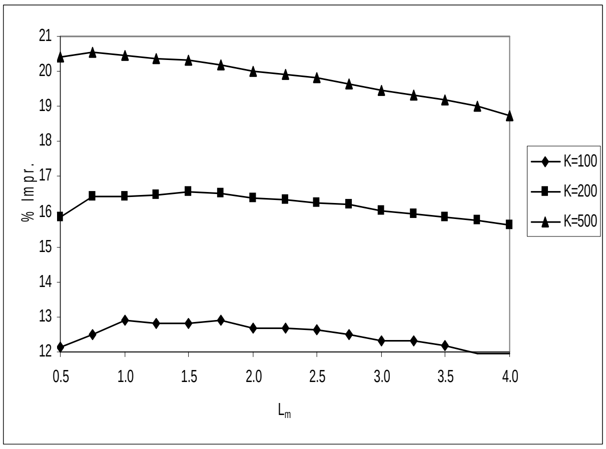


Table 5.5: Pure JRP Savings - $\pi_m = 50$, $\pi'_m = 0$, h=6

	K=100			K=200			K=500		
L_m	(r,Q)	$(Q,\mathbf{S})$	%	(r,Q)	$(Q,\mathbf{S})$	%	(r,Q)	$(Q,\mathbf{S})$	%
0.5	177.636	156.068	12.142	236.794	199.254	15.853	357.267	284.370	20.404
0.75	182.795	159.931	12.508	241.797	202.076	16.428	360.281	286.252	20.548
1	187.716	163.480	12.911	244.854	204.644	16.422	362.346	288.265	20.445
1.25	191.013	166.496	12.835	247.978	207.111	16.480	364.132	289.922	20.380
1.5	194.459	169.541	12.814	250.929	209.395	16.552	365.801	291.465	20.321
1.75	197.814	172.265	12.916	253.421	211.543	16.525	367.258	293.152	20.178
2	200.312	174.902	12.685	255.511	213.692	16.367	368.661	294.972	19.988
2.25	203.017	177.276	12.679	257.672	215.581	16.335	370.009	296.261	19.932
2.5	205.764	179.767	12.634	259.863	217.602	16.263	371.234	297.695	19.809
2.75	207.867	181.901	12.492	261.669	219.321	16.184	372.451	299.267	19.649
3	209.971	184.139	12.303	263.389	221.178	16.026	373.483	300.743	19.476
3.25	212.325	186.213	12.298	265.056	222.829	15.931	374.414	301.973	19.348
3.5	214.241	188.178	12.166	266.821	224.505	15.859	375.370	303.340	19.189
3.75	216.073	190.257	11.948	268.419	226.151	15.747	376.310	304.838	18.993
4	218.015	191.999	11.933	269.770	227.659	15.610	377.202	306.462	18.754

In Table 5.5, we fix the following parameters: $\pi_m = 50$, $\pi'_m = 0$, $h_m = 6$ and $K = 100, 200, 500$. We iterate L_m from 0.5 to 4 by 0.25 units at each step. Figure 5.5

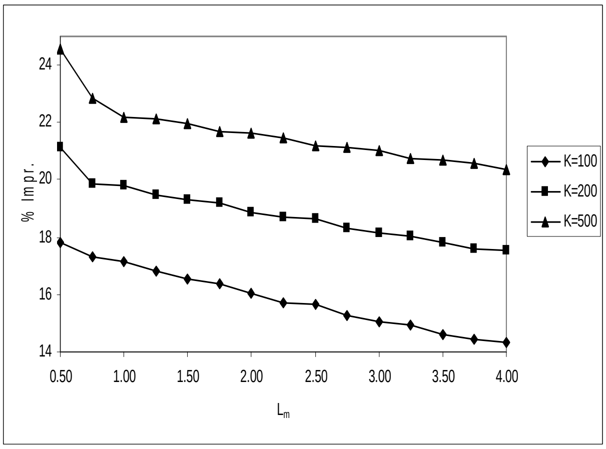
demonstrates the change in percentage savings. In all cases $(Q, \mathbf{S})$ policy provided a smaller channel costs.

However the impact of leadtime on percentage savings through joint replenishment is rather marginal. Even when the leadtime is increased in the range of 8 times, percentage savings differ at most 1%. Typically, as L_m decreases, the percentage improvement through joint replenishment decreases. The only exception is when the lead times are increased from very small values to values around 1. When $K = 100$, a small increase is observed when L_m is increase to 1 from 0.5. The reason for this distortion is at small leadtimes, holding cost savings brought by $(Q, \mathbf{S})$ dominates the increase in backorder costs caused by the increase in lead times and deviation between (r, Q) and $(Q, \mathbf{S})$ is increased. But for larger leadtimes increase in backorder costs diminish the total improvement.

Table 5.6: Pure JRP Savings - $\pi_m = 0$, $\pi'_m = 50$, h=6

	K=100			K=200			K=500		
L_m	(r, Q)	$(Q, \mathbf{S})$	%	(r, Q)	$(Q, \mathbf{S})$	%	(r, Q)	$(Q, \mathbf{S})$	%
0.5	155.772	127.986	17.837	216.360	170.701	21.103	340.368	256.736	24.571
0.75	159.381	131.764	17.328	216.844	173.753	19.872	335.565	258.907	22.844
1	163.114	135.167	17.134	220.286	176.683	19.794	335.976	261.447	22.183
1.25	166.684	138.611	16.842	222.788	179.356	19.494	338.297	263.500	22.110
1.5	169.561	141.556	16.516	225.465	181.982	19.286	340.226	265.571	21.943
1.75	172.869	144.596	16.355	228.408	184.569	19.193	342.104	267.993	21.663
2	175.574	147.385	16.055	230.595	187.037	18.889	344.153	269.806	21.603
2.25	178.280	150.266	15.714	232.948	189.406	18.692	346.191	271.800	21.488
2.5	181.190	152.845	15.644	235.591	191.728	18.618	347.804	274.130	21.183
2.75	183.539	155.491	15.282	237.535	194.101	18.285	349.652	275.749	21.136
3	185.929	157.911	15.069	239.756	196.195	18.169	351.622	277.682	21.028
3.25	188.677	160.523	14.922	242.084	198.439	18.029	353.151	279.823	20.764
3.5	190.645	162.760	14.627	243.948	200.522	17.801	354.821	281.397	20.693
3.75	193.009	165.138	14.440	245.930	202.616	17.612	356.591	283.275	20.560
4	195.358	167.359	14.332	248.141	204.585	17.553	358.210	285.258	20.366

Figure 5.6: Pure JRP Savings - $\pi_m = 0$, $\pi'_m = 50$, h=6

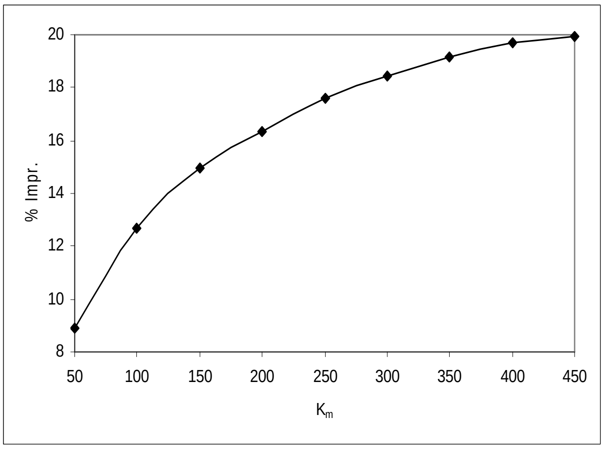


In Table 5.6, we fix the following parameters: $\pi_m = 0$, $\pi'_m = 50$, $h_m = 6$ and $K = 100, 200, 500$. Again we iterate L_m from 0.5 to 4 by 0.25 units at each step. As we did before, we change the backorder type and examine the situation. Figure 5.6 demonstrates the change in percentage savings. The behavior is very similar to previous case but this time the diminishing effect of increased backorders is seen more clearly.

Table 5.7: Pure JRP Savings - $\pi_m = 50$, $\pi'_m = 0$, $h_m = 6$, $L_m = 2$

K	%
50	8.924
100	12.685
150	14.974
200	16.367
250	17.591
300	18.432
350	19.141
400	19.672
450	19.937

Figure 5.7: Pure JRP Savings - $\pi_m = 50$, $\pi'_m = 0$, $h_m = 6$, $L_m = 2$

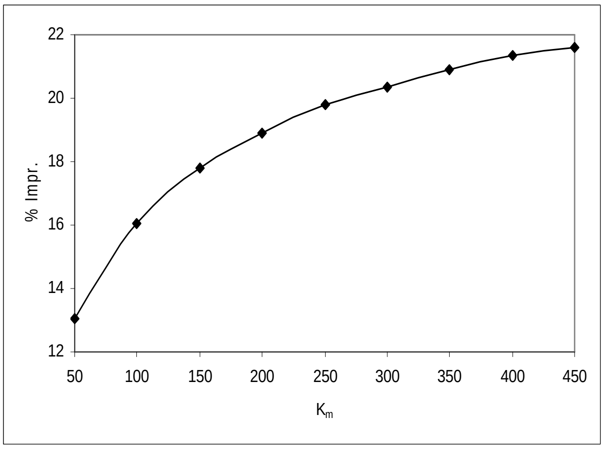


In Table 5.7, we fix the following parameters: $\pi_m = 50$, $\pi'_m = 0$, $h_m = 6$, $L_m = 2$ and this time we iterate K from 50 to 450 by 50 units at each step. Figure 5.7 demonstrates the change in percentage savings. It is observed that as setup cost increases, savings achieved from joint replenishment increases, however there are diminishing marginal returns of percentage savings.

Table 5.8: Pure JRP Savings - $\pi_m = 0$, $\pi'_m = 50$, $h_m = 6$, $L_m = 2$

K	%
50	13.036
100	16.055
150	17.783
200	18.889
250	19.803
300	20.357
350	20.885
400	21.338
450	21.622

Figure 5.8: Pure JRP Savings - $\pi_m = 0$, $\pi'_m = 50$, $h_m = 6$, $L_m = 2$



In Table 5.8, we fix the following parameters: $\pi_m = 50$, $\pi'_m = 0$, $h_m = 6$, $L_m = 2$ and this time we iterate K from 50 to 450 by 50 units at each step. As we did before, we change the backorder type and examine the situation. Figure 5.8 demonstrates the change in percentage savings. As expected average difference increases as setup cost increases. Slightly larger savings are achieved when compared to previous case.

5.2 Physical Improvement Under Centralized Control

In this section we demonstrate the savings achieved through physical improvement and joint replenishment together in various situations. For this reason we have constructed 12 base cases. The base case parameters and optimal solutions of base cases which define the before contract setting are given in Table 5.9 and Table 5.10. Parameters given in Table 5.9 are of a single retailer only. Before contract, the retailer uses (r, Q) policy to manage inventories of her installations. For the sake of simplicity we assume that, the retailer has two identical installations. So channel cost before contract is two times the total cost of a retailer installation. After contract, the manufacturer assumes the control. Under centralized control, the backorder cost parameters that the manufacturer observes are exactly equal to the backorder cost parameters that the retailer sees. In other words, the retailer truly reflects its own backorder costs under centralized control and the retailer and the manufacturer act as a single entity. The channel cost after contract simply equals to the manufacturer's cost.

Table 5.9: Contracts with Setup - Base Case Parameter Summary

Base Case	L_r	h_r	K	π_r	π'_r
1	2	6	100	50	0
2	2	6	200	50	0
3	2	6	500	50	0
4	2	6	100	100	0
5	2	6	200	100	0
6	2	6	500	100	0
7	2	6	100	0	50
8	2	6	200	0	50
9	2	6	500	0	50
10	2	6	100	0	100
11	2	6	200	0	100
12	2	6	500	0	100

Table 5.10: Contracts with Setup - Base Case Solution Summary

Base Cases Before Contract							Cost Components				Costs	
Case	Q	r	K	h_r	π'_r	π_r	Setup	Holding	BO_2	BO_1	Retailer	Channel
1	15	11	100	6	0	50	33.333	54.500	0	12.323	100.156	200.312
2	21	10	200	6	0	50	47.620	66.596	0	13.540	127.756	255.511
3	32	8	500	6	0	50	78.125	87.962	0	18.244	184.331	368.661
4	15	13	100	6	0	100	33.333	66.159	0	9.013	108.505	217.010
5	20	12	200	6	0	100	50	75.216	0	11.450	136.666	273.332
6	31	11	500	6	0	100	80.645	102.242	0	11.925	194.812	389.624
7	15	9	100	6	50	0	33.333	43.334	11.120	0	87.787	175.574
8	21	8	200	6	50	0	47.620	55.465	12.212	0	115.297	230.595
9	32	6	500	6	50	0	78.125	77.031	16.921	0	172.077	344.153
10	16	10	100	6	100	0	31.250	51.782	13.030	0	96.062	192.124
11	20	10	200	6	100	0	50	63.625	10.424	0	124.049	248.099
12	31	9	500	6	100	0	80.645	90.646	10.761	0	182.052	364.103

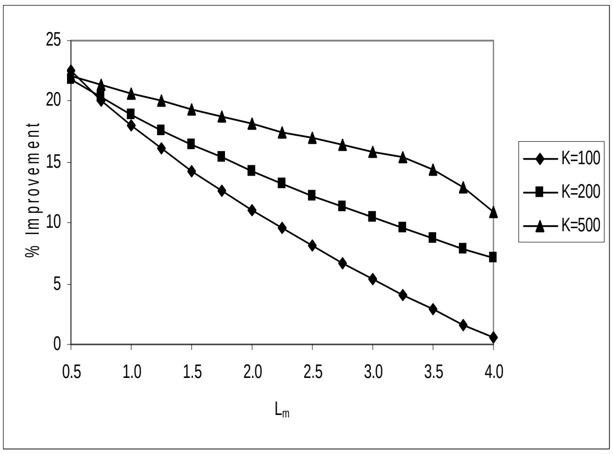
As we mentioned before, physical improvement consists of holding cost reduction and leadtime improvement. We analyze the savings achieved through leadtime reduction and joint replenishment in Section 5.2.1, and through holding cost reduction and joint replenishment, in Section 5.2.2.

5.2.1 Contracts With Setup Cost - Leadtime Improvement

Table 5.11: Contracts with Setup - Leadtime Improvement, $\pi_m = 50$, $\pi'_m = 0$, Case:1,2,3

	K=100				K=200				K=500			
L_m	Q_m	S	Total	% Impr.	Q_m	S	Total	% Impr.	Q_m	S	Total	% Impr.
0.5	19	14	156.068	22.088	28	18	199.254	22.017	40	23	284.370	22.864
0.75	20	16	159.931	20.159	28	19	202.076	20.913	40	24	286.252	22.354
1	20	17	163.480	18.387	29	21	204.644	19.908	40	25	288.265	21.808
1.25	21	19	166.496	16.881	28	22	207.111	18.943	40	27	289.922	21.358
1.5	20	20	169.541	15.362	30	24	209.395	18.049	40	28	291.465	20.940
1.75	21	22	172.265	14.001	29	25	211.543	17.208	40	29	293.152	20.482
2	21	23	174.902	12.685	31	27	213.692	16.367	40	31	294.972	19.988
2.25	22	25	177.276	11.500	30	28	215.581	15.628	40	32	296.261	19.639
2.5	22	26	179.767	10.256	30	29	217.602	14.836	40	33	297.695	19.250
2.75	23	28	181.901	9.191	31	31	219.321	14.164	40	34	299.267	18.823
3	22	29	184.139	8.074	31	32	221.178	13.437	40	36	300.743	18.423
3.25	24	31	186.213	7.038	32	34	222.829	12.791	40	37	301.973	18.089
3.5	23	32	188.178	6.058	31	35	224.505	12.135	40	38	303.340	17.719
3.75	24	34	190.257	5.019	33	37	226.151	11.491	40	39	304.838	17.312
4	24	35	191.999	4.150	32	38	227.659	10.901	40	40	306.462	16.872

Figure 5.9: Contracts with Setup - Leadtime Improvement, $\pi_m = 50$, $\pi'_m = 0$

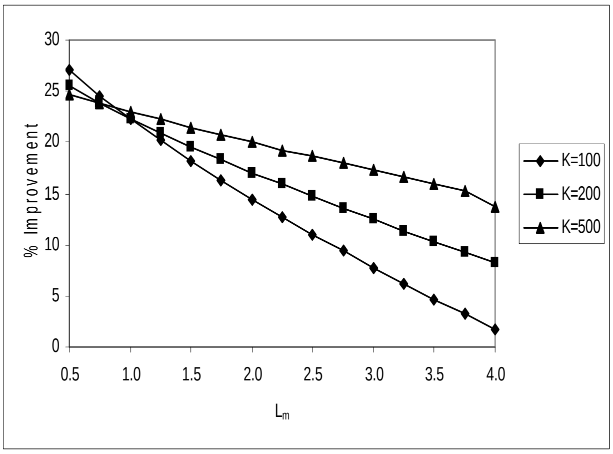


In Table 5.11 we summarize savings obtained from leadtime improvement in base case 1,2 and 3. We iterate L_m from 0.5 to 4. We present optimal Q and S values of the system where S is the base stock level of a single retailer installation under $(Q, \mathbf{S})$ policy. The analysis is done for $K = 100, 200, 500$ and it is observed that the most savings are achieved when $K = 500$. As we have seen before, further reductions in leadtime through consignment contract result in more savings. An interesting observation is, even if the manufacturer leadtime is as large as two times the retailer leadtime, savings are still possible. Results are also demonstrated in Figure 5.9.

Table 5.12: Contracts with Setup - Leadtime Improvement, $\pi_m = 100$, $\pi'_m = 0$

	K=100				K=200				K=500			
L_m	Q_m	S	Total	% Impr.	Q_m	S	Total	% Impr.	Q_m	S	Total	% Impr.
0.5	18	15	168.257	22.466	25	18	213.780	21.787	40	25	303.368	22.138
0.75	19	17	173.363	20.113	26	20	217.764	20.330	40	26	306.580	21.314
1	18	18	178.047	17.955	27	22	221.520	18.956	40	28	309.095	20.668
1.25	19	20	182.145	16.066	26	23	225.111	17.642	40	29	311.627	20.019
1.5	21	22	186.113	14.237	28	25	228.316	16.469	40	30	314.554	19.267
1.75	20	23	189.607	12.628	27	26	231.402	15.340	40	32	316.560	18.753
2	21	25	193.026	11.052	28	28	234.207	14.314	40	33	318.911	18.149
2.25	20	26	196.330	9.530	29	30	237.103	13.255	40	35	321.431	17.502
2.5	21	28	199.401	8.114	29	31	239.788	12.272	40	36	323.297	17.023
2.75	20	29	202.569	6.654	30	33	242.371	11.327	40	37	325.497	16.459
3	22	31	205.305	5.394	29	34	244.750	10.457	40	39	327.722	15.888
3.25	21	32	208.096	4.108	30	36	247.277	9.532	40	40	329.483	15.436
3.5	22	34	210.677	2.918	30	37	249.528	8.709	39	40	333.374	14.437
3.75	21	35	213.394	1.666	29	38	251.817	7.871	36	40	339.256	12.927
4	23	37	215.829	0.544	30	40	253.961	7.087	34	40	347.045	10.928

Figure 5.10: Contracts with Setup - Leadtime Improvement, $\pi_m = 100$, $\pi'_m = 0$



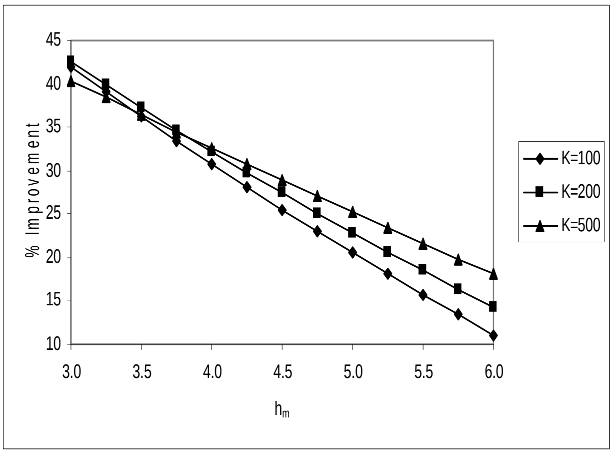
In Table 5.12 we summarize savings obtained from leadtime improvement in base case 4,5 and 6. The difference between these base cases and previous ones is the greater backorder. Again we iterate L_m from 0.5 to 4. Same results with the previous cases are obtained. However with larger backorder costs, the savings diminish faster as it can be observed in Figure 5.10.

Table 5.13: Contracts with Setup - Leadtime Improvement, $\pi_m = 0$, $\pi m' = 50$

	K=100				K=200				K=500			
L_m	Q_m	S	Total	% Impr.	Q_m	S	Total	% Impr.	Q_m	S	Total	% Impr.
0.5	20	12	127.986	27.104	27	15	170.701	25.974	40	21	256.736	25.401
0.75	19	13	131.764	24.952	28	17	173.753	24.650	40	22	258.907	24.770
1	20	15	135.167	23.014	28	18	176.683	23.379	40	23	261.447	24.032
1.25	21	17	138.611	21.053	29	20	179.356	22.220	40	25	263.500	23.435
1.5	21	18	141.556	19.375	28	21	181.982	21.082	40	26	265.571	22.834
1.75	22	20	144.596	17.644	29	23	184.569	19.960	40	27	267.993	22.130
2	21	21	147.385	16.055	29	24	187.037	18.889	40	29	269.806	21.603
2.25	22	23	150.266	14.415	30	26	189.406	17.862	40	30	271.800	21.023
2.5	22	24	152.845	12.945	29	27	191.728	16.855	40	31	274.130	20.347
2.75	23	26	155.491	11.438	30	29	194.101	15.826	40	33	275.749	19.876
3	22	27	157.911	10.060	30	30	196.195	14.918	40	34	277.682	19.314
3.25	23	29	160.523	8.572	31	32	198.439	13.945	40	36	279.823	18.692
3.5	23	30	162.760	7.298	30	33	200.522	13.041	40	37	281.397	18.235
3.75	22	31	165.138	5.944	30	34	202.616	12.133	40	38	283.275	17.689
4	23	33	167.359	4.679	31	36	204.585	11.279	40	40	285.258	17.113

In Table 5.13 we summarize savings obtained from leadtime improvement in base

Figure 5.11: Contracts with Setup - Leadtime Improvement, $\pi_m = 0$, $\pi'_m = 50$



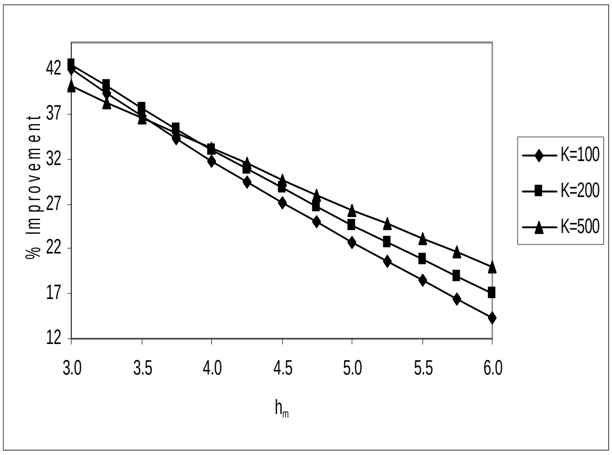
case 7,8 and 9. For these base cases, we have type II backorder costs, as opposed to type I backorder costs in the previous cases. Again we iterate L_m from 0.5 to 4. Same results with the previous cases are obtained. Results can be observed in 5.11.

Table 5.14: Contracts with Setup - Leadtime Improvement, $\pi_m = 0$, $\pi'_m = 100$

	K=100				K=200				K=500			
L_m	Q_m	S	Total	% Impr.	Q_m	S	Total	% Impr.	Q_m	S	Total	% Impr.
0.5	19	13	140.213	27.019	26	16	184.911	25.469	39	22	274.469	24.618
0.75	18	14	144.983	24.536	27	18	188.864	23.875	40	24	277.227	23.860
1	19	16	149.249	22.316	26	19	192.605	22.368	40	25	280.510	22.959
1.25	20	18	153.404	20.153	27	21	196.074	20.969	40	27	283.096	22.248
1.5	19	19	157.317	18.117	28	23	199.542	19.571	40	28	285.861	21.489
1.75	20	21	160.968	16.217	28	24	202.750	18.279	40	30	288.756	20.694
2	20	22	164.460	14.399	29	26	205.862	17.024	40	31	291.074	20.057
2.25	21	24	167.736	12.694	28	27	208.729	15.869	40	32	293.934	19.272
2.5	20	25	171.091	10.947	29	29	211.661	14.687	40	34	296.192	18.652
2.75	21	27	174.098	9.382	28	30	214.458	13.559	40	35	298.616	17.986
3	22	29	177.268	7.732	29	32	217.263	12.429	40	37	301.245	17.264
3.25	21	30	180.159	6.228	29	33	219.916	11.360	40	38	303.281	16.705
3.5	22	32	183.172	4.659	30	35	222.522	10.309	40	39	305.788	16.016
3.75	22	33	185.843	3.269	29	36	224.993	9.313	39	40	308.678	15.222
4	23	35	188.667	1.799	30	38	227.556	8.280	37	40	313.957	13.772

In Table 5.14 we summarize savings obtained from leadtime improvement in base case 10,11 and 12. The difference between these base cases and previous ones is the greater backorder cost. Again we iterate L_m from 0.5 to 4. Same results with the previous cases are obtained. Results can be observed in Figure 5.12. Considering

Figure 5.12: Contracts with Setup - Leadtime Improvement, $\pi_m = 0$, $\pi'_m = 100$



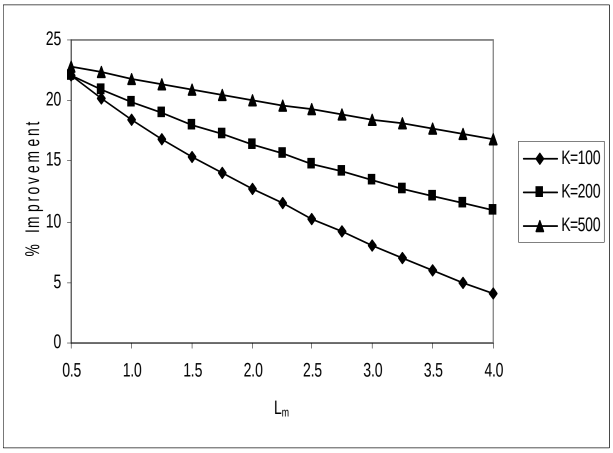
all base cases, greater savings are obtained from leadtime improvement and joint replenishment when $K = 500$. Another common observation is, savings percentage diminish faster under high backorder cost.

5.2.2 Contracts With Setup Cost - Holding cost improvement

Table 5.15: Contracts with Setup - Holding Cost Improvement, $\pi_m = 50$, $\pi'_m = 0$

	K=100				K=200				K=500			
h_m	Q	S	Total	% Impr.	Q	S	Total	% Impr.	Q	S	Total	% Impr.
3	28	28	117.104	41.539	39	33	146.943	42.491	40	33	221.956	39.794
3.25	26	27	122.747	38.722	38	32	153.441	39.948	40	33	228.613	37.988
3.5	27	27	128.087	36.056	36	31	159.765	37.472	40	33	235.270	36.183
3.75	25	26	133.313	33.447	36	31	165.923	35.062	40	32	241.768	34.420
4	25	26	138.341	30.937	35	30	171.752	32.781	40	32	247.943	32.745
4.25	25	26	143.369	28.427	33	29	177.522	30.523	40	32	254.117	31.070
4.5	24	25	148.140	26.045	33	29	183.064	28.354	40	32	260.291	29.396
4.75	24	25	152.802	23.718	32	28	188.553	26.206	40	32	266.465	27.721
5	24	25	157.464	21.391	32	28	193.731	24.179	40	31	272.179	26.171
5.25	22	24	161.919	19.166	32	28	198.909	22.153	40	31	277.878	24.625
5.5	22	24	166.332	16.963	30	27	203.938	20.185	40	31	283.576	23.080
5.75	23	24	170.669	14.798	30	27	208.867	18.255	40	31	289.274	21.534
6	21	23	174.902	12.685	31	27	213.692	16.367	40	31	294.972	19.988

Figure 5.13: Contracts with Setup - Holding Cost Improvement, $\pi_m = 50$, $\pi'_m = 0$

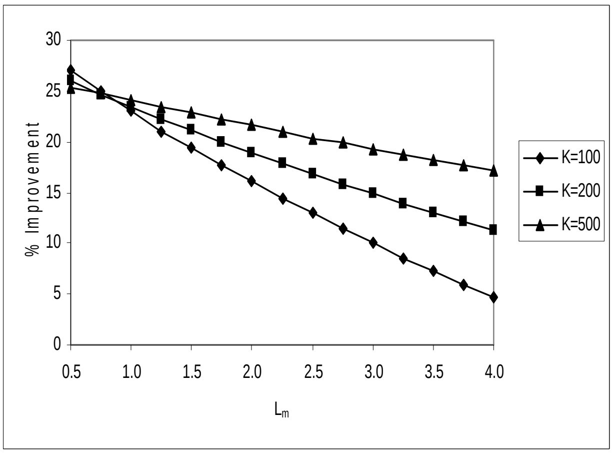


In Table 5.15 we summarize savings obtained from leadtime improvement in base case 1,2 and 3. We iterate h_m from 3 to 6. We present optimal Q and S values of the system where S is the base stock level of a single retailer installation under $(Q, \mathbf{S})$ policy. The analysis is done for $K = 100, 200, 500$ and it is observed that the most remarkable savings are achieved when $K = 200$ but we should note that savings are very close for all setup costs. As we have seen before, reduction in leadtime results in further savings. Improving holding cost to 3 from 6 brings around 40% savings in all setup costs. Results are also demonstrated in Figure 5.13.

Table 5.16: Contracts with Setup - Holding Cost Improvement, $\pi_m = 100$, $\pi'_m = 0$

	K=100				K=200				K=500			
h	Q_m	S	Total	% Impr.	Q_m	S	Total	% Impr.	Q_m	S	Total	% Impr.
3	26	29	126.008	41.934	37	34	156.944	42.581	40	35	232.238	40.394
3.25	27	29	132.313	39.029	36	33	164.390	39.857	40	35	239.875	38.434
3.5	25	28	138.397	36.225	36	33	171.528	37.246	40	35	247.513	36.474
3.75	25	28	144.407	33.456	34	32	178.459	34.710	40	35	255.150	34.514
4	23	27	150.331	30.726	32	31	185.338	32.193	40	34	262.748	32.564
4.25	24	27	156.006	28.111	33	31	191.891	29.796	40	34	269.893	30.730
4.5	22	26	161.635	25.517	31	30	198.315	27.446	40	34	277.038	28.896
4.75	22	26	167.023	23.034	31	30	204.581	25.153	40	34	284.184	27.062
5	22	26	172.411	20.552	31	30	210.847	22.860	40	34	291.329	25.228
5.25	22	26	177.799	18.069	29	29	216.884	20.652	40	34	298.474	23.394
5.5	21	25	182.991	15.676	30	29	222.792	18.490	40	33	305.597	21.566
5.75	21	25	188.008	13.364	28	28	228.561	16.380	40	33	312.254	19.858
6	21	25	193.026	11.052	28	28	234.207	14.314	40	33	318.911	18.149

Figure 5.14: Contracts with Setup - Holding Cost Improvement, $\pi_m = 100$, $\pi'_m = 0$

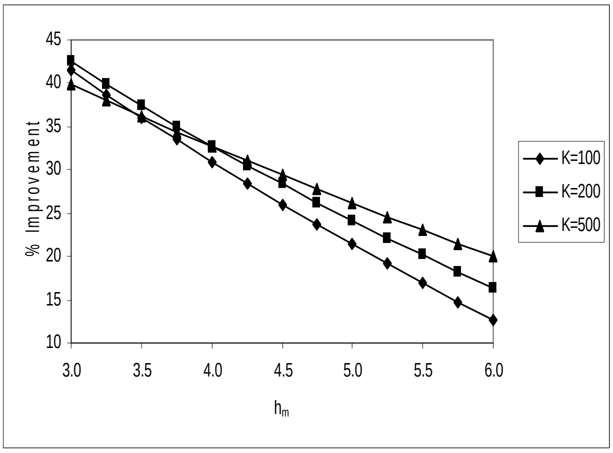


In Table 5.16 we summarize savings obtained from leadtime improvement in base case 4,5 and 6. We iterate h_m from 3 to 6. Similar results are found as previous cases. Results are also demonstrated in Figure 5.14.

Table 5.17: Contracts with Setup - Holding Cost Improvement, $\pi_m = 0$, $\pi'_m = 50$

	K=100				K=200				K=500			
h_m	Q_m	S	Total	% Impr.	Q_m	S	Total	% Impr.	Q_m	S	Total	% Impr.
3	29	26	102.931	41.375	38	30	132.984	42.330	40	31	208.037	39.551
3.25	27	25	107.275	38.900	38	30	138.434	39.967	40	31	213.735	37.895
3.5	27	25	111.595	36.440	36	29	143.664	37.699	40	31	219.434	36.240
3.75	25	24	115.687	34.109	35	28	148.626	35.547	40	30	224.773	34.688
4	26	24	119.664	31.844	35	28	153.470	33.446	40	30	230.005	33.168
4.25	24	23	123.410	29.710	33	27	158.109	31.434	40	30	235.236	31.648
4.5	24	23	127.128	27.593	33	27	162.706	29.441	40	30	240.468	30.128
4.75	24	23	130.845	25.475	32	26	166.978	27.588	40	30	245.699	28.608
5	23	22	134.312	23.501	32	26	171.222	25.748	40	29	250.702	27.154
5.25	23	22	137.679	21.583	30	25	175.322	23.970	40	29	255.478	25.766
5.5	23	22	141.046	19.665	30	25	179.320	22.236	40	29	260.254	24.379
5.75	21	21	144.262	17.834	30	25	183.319	20.502	40	29	265.030	22.991
6	21	21	147.385	16.055	29	24	187.037	18.889	40	29	269.806	21.603

Figure 5.15: Contracts with Setup - Holding Cost Improvement, $\pi_m = 0$, $\pi'_m = 50$

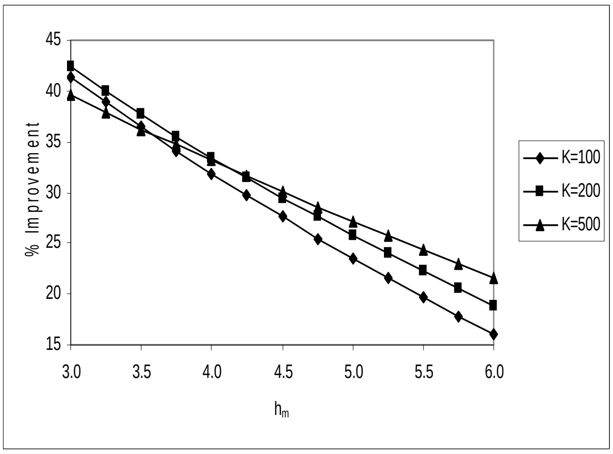


In Table 5.17 we summarize savings obtained from leadtime improvement in base case 7,8 and 9. We iterate h_m from 3 to 6. Similar results are found as previous cases. Results are also demonstrated in Figure 5.15.

Table 5.18: Contracts with Setup - Holding Cost Improvement, $\pi_m = 0$, $\pi'_m = 100$

	K=100				K=200				K=500			
h_m	Q_m	S	Total	% Impr.	Q_m	S	Total	% Impr.	Q_m	S	Total	% Impr.
3	27	27	111.407	42.013	38	32	142.463	42.578	40	33	217.709	40.207
3.25	26	26	116.591	39.314	36	31	148.688	40.069	40	33	224.366	38.379
3.5	26	26	121.502	36.758	35	30	154.691	37.650	40	33	231.023	36.550
3.75	24	25	126.288	34.267	35	30	160.482	35.315	40	32	237.265	34.836
4	24	25	130.950	31.841	33	29	166.027	33.080	40	32	243.439	33.140
4.25	24	25	135.612	29.414	31	28	171.568	30.847	40	32	249.613	31.444
4.5	23	24	139.919	27.173	32	28	176.805	28.736	40	32	255.787	29.749
4.75	23	24	144.216	24.936	30	27	181.930	26.671	40	32	261.962	28.053
5	21	23	148.475	22.719	30	27	186.859	24.684	40	32	268.136	26.357
5.25	21	23	152.525	20.611	30	27	191.788	22.697	40	31	273.979	24.752
5.5	21	23	156.574	18.503	28	26	196.602	20.757	40	31	279.677	23.187
5.75	21	23	160.624	16.395	28	26	201.283	18.870	40	31	285.376	21.622
6	20	22	164.460	14.399	29	26	205.862	17.024	40	31	291.074	20.057

Figure 5.16: Contracts with Setup - Holding Cost Improvement, $\pi_m = 0$, $\pi'_m = 100$



In Table 5.18 we summarize savings obtained from leadtime improvement in base case 10,11 and 12. We iterate h_m from 3 to 6. Similar results are found as previous cases. Results are also demonstrated in Figure 5.16.

Similar to our study in Section 4.1, we compared the effects of leadtime reduction and holding cost reduction. In our data sets, again holding cost reduction brought more savings than leadtime improvement. As holding cost is reduced, inventory levels increase while cost of holding such large inventories decrease which in turn reduces the backorders due to decreased number of stockouts. Similar to what we have found in Section 4.1 as base stock levels decrease due to shorter leadtime, the backorders may increase and hamper the savings.

5.3 Decentralized Control

In this section, we analyze the effect of retailer charging different backorder costs than what she observes on supply chain costs when manufacturer utilizes joint replenishment without providing physical improvement. As we defined in Chapter 3, the retailer observes π_r and π'_r but in the contract she may charge the manufacturer backorder costs which are different (i.e. $\pi_m \neq \pi_r$ and $\pi'_m \neq \pi'_r$). This manipulation can be done in various ways. First the amount the backorder cost may be changed without changing the type of backorder. For example, if retailer is charged per occasion basis by customer, she may charge the manufacturer on per occasion basis but with a larger cost. Second, the retailer may charge a different type of backorder cost (possibly with a different amount than what she faces) to manufacturer, such as incurring Type II backorder cost while facing Type I backorder cost. Again we define 12 base cases to demonstrate the behavior of cost functions of the retailer, the manufacturer and the supply chain. The base case initial parameters for a single retailer and backorder cost ranges that are incurred to manufacturer are given in the Table 5.19. The optimal solution of base cases before contract are given in Table 5.20.

Table 5.19: Contracts with Setup, Decentralized Control - Base Case Parameter Summary

	Retailer					Manufacturer				
Base Case	L_r	h_r	K	π_r	π'_r	L_m	h_m	K	π_m	π'_m
1	2	6	100	0	100	2	6	100	0	[10, 150]
2	2	6	200	0	100	2	6	200	0	[10, 150]
3	2	6	500	0	100	2	6	500	0	[10, 150]
4	2	6	100	0	50	2	6	100	0	[25, 75]
5	2	6	200	0	50	2	6	200	0	[25, 75]
6	2	6	500	0	50	2	6	500	0	[25, 75]
7	2	6	100	50	0	2	6	100	[25, 75]	0
8	2	6	200	50	0	2	6	200	[25, 75]	0
9	2	6	500	50	0	2	6	500	[25, 75]	0
10	2	6	100	0	50	2	6	100	[25, 75]	0
11	2	6	200	0	50	2	6	200	[25, 75]	0
12	2	6	500	0	50	2	6	500	[25, 75]	0

Table 5.20: Contracts with Setup, Decentralized Control - Base Case Solution Summary

Retailer's Optimal Parameters							Cost Components				Costs	
Case	Q_r	r_r	K	h_r	π_r	π'_r	Setup	Holding	BO_1	BO_2	Single Ret	Channel Cost
1	16	10	100	6	0	100	62.500	103.564	0	26.060	96.062	192.124
2	20	10	200	6	0	100	100	127.251	0	20.848	124.049	248.099
3	31	9	500	6	0	100	161.290	181.291	0	21.522	182.052	364.103
4	15	9	100	6	0	50	66.666	86.669	0	22.239	87.787	175.574
5	21	8	200	6	0	50	95.240	110.931	0	24.424	115.297	230.595
6	32	6	500	6	0	50	156.250	154.061	0	33.842	172.077	344.153
7	15	11	100	6	50	0	66.666	109.001	24.645	0	100.156	200.312
8	21	10	200	6	50	0	95.240	133.191	27.080	0	127.756	255.511
9	32	8	500	6	50	0	156.250	175.923	36.488	0	184.331	368.661
10	15	9	100	6	0	50	66.666	86.669	0	22.239	87.787	175.574
11	21	8	200	6	0	50	95.240	110.931	0	24.424	115.297	230.595
12	32	6	500	6	0	50	156.250	154.061	0	33.842	172.077	344.153

Table 5.21: Contracts with Setup, Decentralized Control, $\pi_m = 0$, $\pi'_m = 10 : 150$, Case: 1, 2, 3

	K=100				K=200				K=500			
π'_m	MFG	RET	CHN	% IMPR.	MFG	RET	CHN	% IMPR.	MFG	RET	CHN	% IMPR.
10	103.41	247.21	350.63	-82.500	136.48	318.68	455.16	-83.460	212.63	341.07	553.70	-52.073
20	122.97	115.38	238.35	-24.061	159.35	125.65	285	-14.872	238.35	154.92	393.27	-8.011
30	134.02	61.38	195.40	-1.705	171.97	70.59	242.56	2.231	252.64	79.03	331.67	8.907
40	141.74	35.58	177.32	7.705	180.57	42.91	223.48	9.924	262.41	50.10	312.52	14.168
50	147.39	24.79	172.18	10.382	187.04	30.42	217.46	12.350	269.81	30.18	299.99	17.609
60	152.11	16.43	168.54	12.277	192.24	16.53	208.76	15.855	275.84	24.15	299.99	17.609
70	155.86	9.57	165.43	13.893	196.37	12.40	208.76	15.855	280.38	12.78	293.16	19.485
80	159.05	6.38	165.43	13.893	199.89	6.85	206.75	16.668	284.64	8.52	293.16	19.485
90	161.87	2.59	164.46	14.399	203.09	2.81	205.91	17.005	288.14	2.93	291.07	20.057
100	164.46	0	164.46	14.399	205.86	0	205.86	17.024	291.07	0	291.07	20.057
110	166.66	-1.99	164.67	14.288	208.18	-2.22	205.96	16.983	294.01	-2.93	291.07	20.057
120	168.65	-3.97	164.67	14.288	210.40	-4.44	205.96	16.983	296.76	-3.93	292.83	19.574
130	170.49	-4.74	165.75	13.727	212.42	-5.37	207.05	16.544	298.73	-5.90	292.83	19.574
140	172.07	-6.32	165.75	13.727	214.22	-7.16	207.05	16.544	300.69	-7.86	292.83	19.574
150	173.65	-7.90	165.75	13.727	216.01	-8.95	207.05	16.544	302.66	-9.83	292.83	19.574

In Table 5.21, we present the costs of the manufacturer, the retailer and the channel in base case 1, 2 and 3, after contract. Again note that the cost figures for the retailer and the manufacturer exclude the annual payment A. Note that setup cost is the only parameter that distinguishes these base cases. Type II backorder cost incurred by retailer, π'_m, is iterated between 10 and 150 by units of 10. It is observed that when the retailer charges a backorder cost that is too low, channel suffers since the manufacturer keeps insufficient inventory. Minimum channel cost and maximum percentage savings are achieved in intervals around original Type II backorder cost, 100. When compared to the results we observed in Chapter 4, here intervals are much more narrow. Greatest percentage savings are achieved under $K = 500$. The percentage savings are depicted in Figure 5.17. In Figure 5.18, the costs of manufacturer, retailer and channel in base case 1 are depicted.

Figure 5.17: Contracts with Setup, Decentralized Control, $\pi_m = 0$, $\pi'_m = 10 : 150$, Case: 1, 2, 3

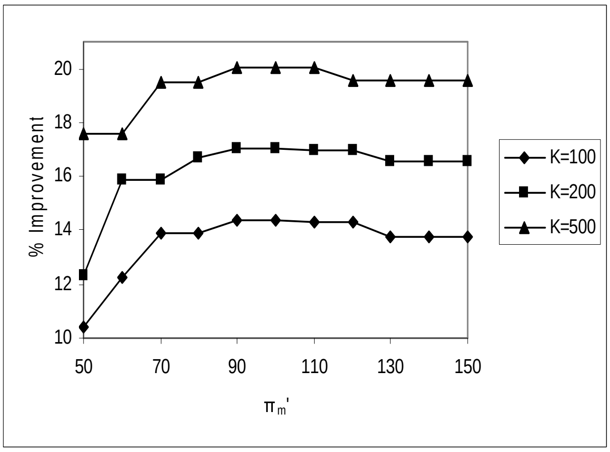


Figure 5.18: Contracts with Setup, Decentralized Control, $\pi_m = 0$, $\pi'_m = 10 : 150$, Case 1 Costs

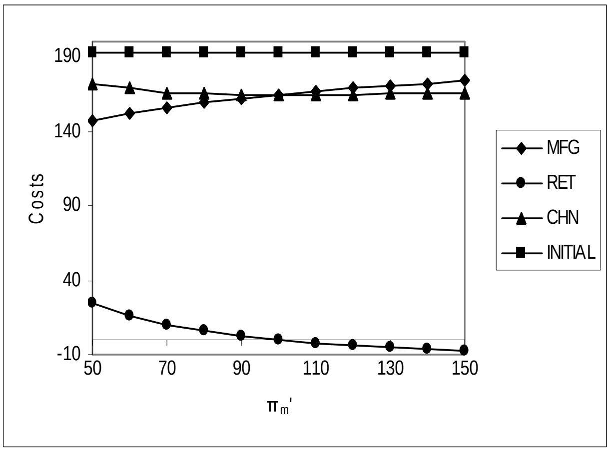


Table 5.22: Contracts with Setup, Decentralized Control, $\pi_m = 0$, $\pi'_m = 25 : 75$, Case:4, 5, 6

	K=100				K=200				K=500			
π'_m	MFG	RET	CHN	% IMPR.	MFG	RET	CHN	% IMPR.	MFG	RET	CHN	% IMPR.
25	129.14	25.54	154.68	11.899	166.43	28.97	195.40	15.262	246.30	37.34	283.65	17.581
30	134.02	17.54	151.56	13.678	171.97	20.17	192.14	16.676	252.64	22.58	275.22	20.029
35	138.23	11.20	149.43	14.893	176.69	13.07	189.76	17.709	258.24	12.53	270.76	21.325
40	141.74	5.93	147.67	15.895	180.57	7.15	187.72	18.594	262.41	8.35	270.76	21.325
45	144.70	2.97	147.67	15.895	183.99	3.04	187.04	18.889	266.59	4.18	270.76	21.325
50	147.39	0	147.39	16.055	187.04	0	187.04	18.889	269.81	0	269.81	21.603
55	149.86	-2.48	147.39	16.055	189.74	-2.57	187.17	18.832	272.82	-3.02	269.81	21.603
60	152.11	-4.11	148	15.704	192.24	-4.13	188.10	18.427	275.84	-6.04	269.81	21.603
65	154.07	-5.83	148.24	15.571	194.30	-6.20	188.10	18.427	278.25	-6.39	271.86	21.007
70	155.86	-6.38	149.48	14.861	196.37	-8.26	188.10	18.427	280.38	-8.52	271.86	21.007
75	157.46	-7.97	149.48	14.861	198.18	-8.57	189.62	17.771	282.51	-10.65	271.86	21.007

In Table **??**, we present the costs of the manufacturer, the retailer and channel in base case 4, 5 and 6, after contract. Type II backorder cost incurred by retailer, π'_m, is iterated between 25 and 75 by units of 5. Different from previous cases, backorder costs are lower. Minimum channel cost and maximum percentage savings are achieved in intervals around original Type II backorder cost, 50. Again, greatest percentage savings are achieved under $K = 500$. The percentage savings are shown in Figure 5.19. In Figure 5.20, the costs of the manufacturer, the retailer and the channel in base case 4 are depicted.

Figure 5.19: Contracts with Setup, Decentralized Control, $\pi_m = 0$, $\pi'_m = 25 : 75$, Case:4, 5, 6

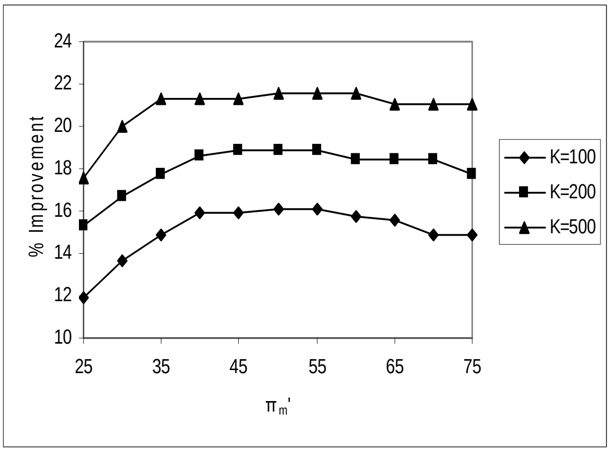


Figure 5.20: Contracts with Setup, Decentralized Control, $\pi_m = 0$, $\pi'_m = 25 : 75$, Case 3 Costs

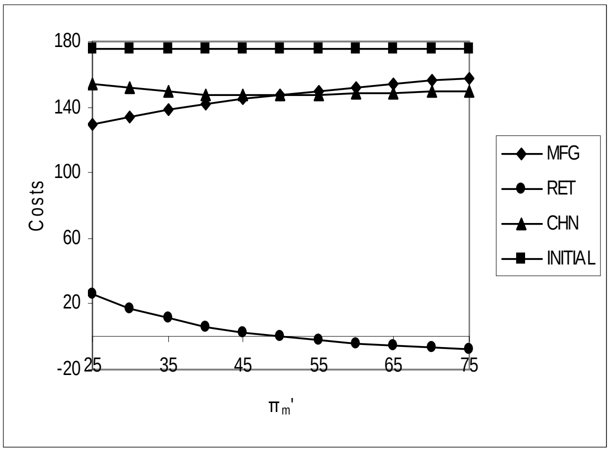


Table 5.23: Contracts with Setup, Decentralized Control, $\pi_m = 25 : 75$, $\pi'_m = 0$, Case:7, 8, 9

	K=100				K=200				K=500			
π_m	MFG	RET	CHN	% IMPR.	MFG	RET	CHN	% IMPR.	MFG	RET	CHN	% IMPR.
25	153.10	33.06	186.15	7.068	187.68	42.29	229.97	9.997	264.32	55.35	319.67	13.289
30	159.31	23.34	182.65	8.818	195.31	27.79	223.09	12.688	273.10	29.39	302.49	17.949
35	164.27	12.41	176.68	11.798	201.21	15.79	217	15.070	280.12	17.35	297.47	19.311
40	168.31	7.09	175.40	12.434	206.05	9.28	215.33	15.725	285.90	11.57	297.47	19.311
45	171.86	3.55	175.40	12.434	210.17	4.06	214.23	16.155	290.53	4.44	294.97	19.988
50	174.90	0	174.90	12.685	213.69	0	213.69	16.367	294.97	0	294.97	19.988
55	177.54	-2.38	175.16	12.557	216.68	-2.88	213.80	16.326	298.29	-3.32	294.97	19.988
60	179.92	-4.76	175.16	12.557	219.42	-4.90	214.52	16.042	301.62	-6.64	294.97	19.988
65	182.01	-5.93	176.09	12.093	221.88	-7.35	214.52	16.042	304.58	-7.25	297.34	19.347
70	183.99	-7.90	176.09	12.093	224.13	-8.25	215.89	15.509	307	-9.66	297.34	19.347
75	185.88	-8.11	177.77	11.252	226.20	-10.31	215.89	15.509	309.41	-12.08	297.34	19.347

In Table 5.23, we present the costs of the manufacturer, the retailer and channel in base case 7, 8 and 9, after contract. Type I backorder cost incurred by retailer, π_m,is iterated between 25 and 75 by units of 5. The difference from previous cases is, now backorder cost type is different. Minimum channel cost and maximum percentage savings are achieved in intervals around original Type I backorder cost, 50. Again, greatest percentage savings are achieved under $K = 500$. The percentage savings are shown in Figure 5.21. In Figure 5.22, the costs of the manufacturer, the retailer and the channel in base case 7 are depicted.

Figure 5.21: Contracts with Setup, Decentralized Control,$\pi_m = 25 : 75$, $\pi'_m = 0$, Case:7, 8, 9

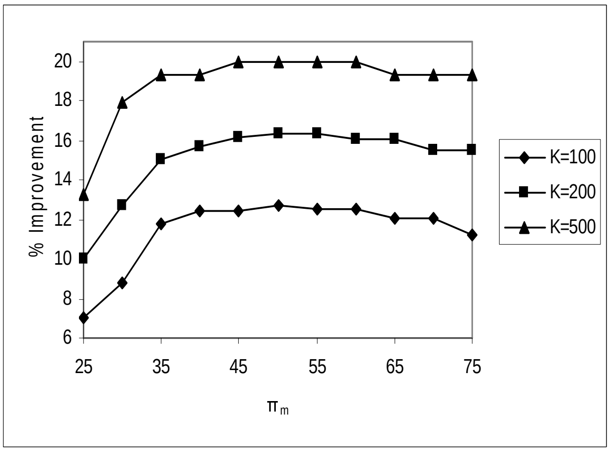


Figure 5.22: Contracts with Setup, Decentralized Control, $\pi_m = 25 : 75$, $\pi'_m = 0$, Case 7 Costs

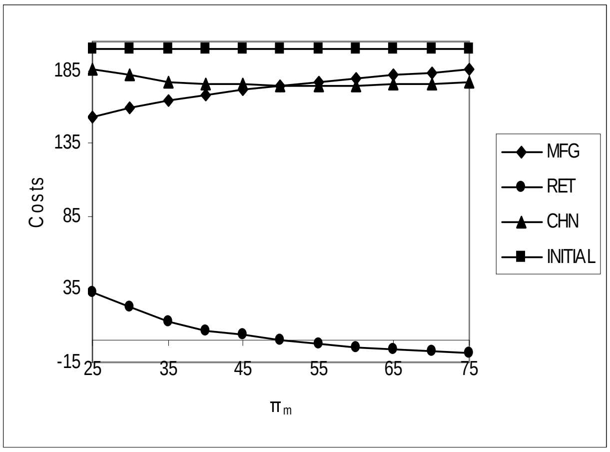


Table 5.24: Contracts with Setup, Decentralized Control, $\pi_m = 25 : 75$, $\pi'_m = 0$, Case:10, 11, 12

	K=100				K=200				K=500			
π_m	MFG	RET	CHN	% IMPR.	MFG	RET	CHN	% IMPR.	MFG	RET	CHN	% IMPR.
25	153.10	-4.96	148.14	15.625	187.68	4.06	191.74	16.851	264.32	19.33	283.65	17.581
30	159.31	-11.53	147.78	15.830	195.31	-7.31	187.99	18.476	273.10	-2.33	270.76	21.325
35	164.27	-13.75	150.51	14.274	201.21	-13.01	188.20	18.385	280.12	-10.31	269.81	21.603
40	168.31	-16.02	152.29	13.261	206.05	-17.17	188.88	18.092	285.90	-16.10	269.81	21.603
45	171.86	-19.57	152.29	13.261	210.17	-19.95	190.23	17.506	290.53	-18.67	271.86	21.007
50	174.90	-20.16	154.74	11.865	213.69	-20.34	193.35	16.150	294.97	-18.56	276.42	19.682
55	177.54	-18.58	158.96	9.462	216.68	-20.91	195.77	15.103	298.29	-21.88	276.42	19.682
60	179.92	-20.95	158.96	9.462	219.42	-20.68	198.74	13.814	301.62	-25.20	276.42	19.682
65	182.01	-19.70	162.32	7.550	221.88	-23.13	198.74	13.814	304.58	-21.58	283.01	17.767
70	183.99	-21.67	162.32	7.550	224.13	-21.89	202.24	12.295	307	-23.99	283.01	17.767
75	185.88	-19.66	166.22	5.329	226.20	-23.95	202.24	12.295	309.41	-26.41	283.01	17.767

In Table 5.24, we present the costs of the manufacturer, the retailer and channel in base case 10, 11 and 12, after contract. Different from previous cases, the retailer charges a different backorder cost type. The retailer faces backorders in per unit per time basis, but charges the manufacturer in per occasion basis. Minimum channel cost and maximum percentage savings are achieved in intervals around 30 for all cases. This is similar to our findings in Chapter 4. Again, greatest percentage savings are achieved under $K = 500$. The percentage savings are shown in Figure 5.23. In Figure 5.24, the costs of the manufacturer, the retailer and the channel in base case 10 are depicted.

Figure 5.23: Contracts with Setup, Decentralized Control, $\pi_m = 25 : 75$, $\pi'_m = 0$, Case:10, 11, 12

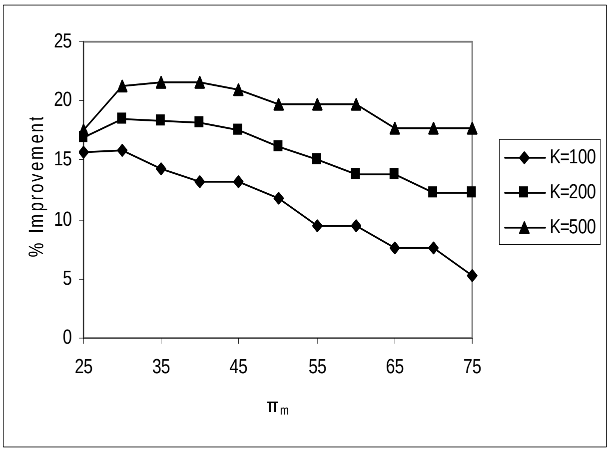
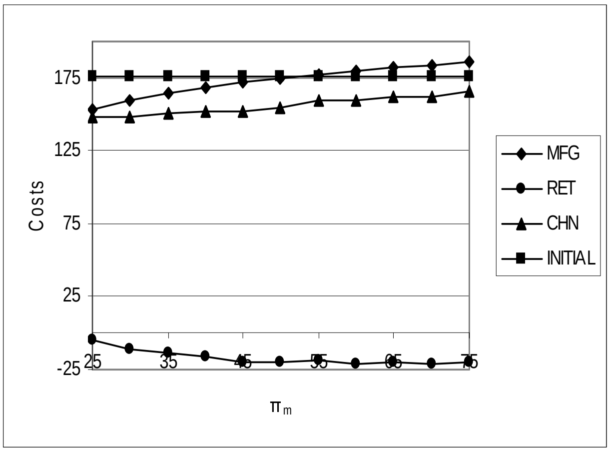


Figure 5.24: Contracts with Setup, Decentralized Control, $\pi_m = 25 : 75$, $\pi'_m = 0$, Case 10 Costs



Chapter 6

Conclusion

In this thesis, we consider a spare parts inventory system. In this system, the manufacturer provides spare parts of a capital equipment to its customers. The manufacturer and its customers agree to a full service Vendor Managed Inventory (VMI) contract to coordinate their activities and exploit the benefits of VMI. The specific contract we will consider is a consignment contract, under which the manufacturer assumes the responsibility and the ownership of the inventory in a stock room inside the facilities of its customers. In exchange for this service, the customers pay an annual fee.

In the setting we consider, moving the control from the customer to the manufacturer can provide system improvements such as lower cost of inventory ownership, shorter leadtime and the ability to jointly replenish multiple installations. We first use basic inventory models to quantify the savings obtained through these improvements. For the case of no setup costs, the customers before the contract and the manufacturer after the contract use a simple base stock policy. For the case of setup costs, the customers before the contract use independent (r, Q) policy at each installation and the manufacturer after the contract uses a $(Q, \mathbf{S})$ policy to jointly manage multiple installations.

There can be various types of contracts which are structured using different terms. Service levels, inventory levels and backorder costs are some examples of possible terms on which the contract can be structured. We structure our contract on backorder costs (π, π') and the annual payment of the delegating party. Using the cost expressions that are introduced in beforehand mentioned models, we conduct a numerical study to demonstrate the savings that are achieved through leadtime and holding cost reduction in a setting without setup costs. It is observed that both leadtime reduction and holding cost reduction are considerably effective. We then examine the impact of the retailer charging backorder costs that are different from what she observes. We show numerically that, if the retailer manipulates backorder penalties, the supply chain efficiency will suffer, and in fact, the supply chain costs may be higher than before the contract even if there are physical improvements mentioned above. We repeat the same analysis for the case of positive setup costs for which similar results are obtained. We also demonstrate the effect of joint replenishment alone by comparing the total costs obtained from (r, Q) and $(Q, \mathbf{S})$ policies. Joint replenishment brings savings in all cases and the savings are the most remarkable under high setup costs. It is also found that as per unit backorder costs increase, the savings through joint replenishment diminish. In our data sets, holding cost reduction brought more savings than leadtime improvement. As holding cost is reduced, inventory levels increase while cost of holding such large inventories decrease which in turn reduces the backorders due to decreased number of stockouts. But in leadtime reduction case, as base stock levels decrease due to shorter leadtime, the backorders may increase and hamper the savings.

We note that the primary difference between our study and earlier research is that we extend the consignment contracts literature in the direction of joint replenishment. We use backorder costs and the annual fee as the terms of the contract and search for values of these variables which coordinate the supply chain. In this research, we use leadtime reduction, holding cost reduction and joint replenishment to create savings in a spare parts consignment contract. However, to our knowledge our study is the first to simultaneously consider these concepts. Our numerical results indicate that simultaneous usage of physical improvement and joint replenishment indeed results in significant inventory and cost savings.

Future research can extend the analysis here in many directions. A natural question to consider is how to allocate those savings to the parties in the supply chain.

In our models, we state that the customers pay an annual fee to the manufacturer for the consignment service. However, we did not elaborate on how to determine this fee, except for giving a range. To specify the exact amount of this fee, bargaining models can be used. Another extension may include quantifying the cost of manufacturer's effort to reduce its leadtime. A further extension could be to use different joint replenishment policies such as $(\mathbf{S}, \mathbf{c}, \mathbf{s})$ policy. Finally, our numerical results could be strengthened by using more than two installation that are non–identical.

Bibliography

[1] Applied Materials Inc. Amat annual report. *www.supplychainonline.com*, 2004.

[2] B. Arnold. E-commerce, increasing device complexity offer new twists in value-added service support. *Semiconductor Magazine*, 1(4), 2000.

[3] A. Aschner. *The Gower Handbook of Logistics and Distribution management*, chapter Managing and Controlling Logistics Inventories. Aldershot: Gower, Brookfield, 1990.

[4] D. R. Atkins and P. O. Iyogun. Periodic versus ćan-orderṕolicies for coordinated multi-item inventory systems. *Management Science*, (34):791–796, 1988.

[5] S. Axster. *Inventory Control*. Kluwer, Boston, 2000.

[6] K. R. Baker. Lot-sizing procedures and a standard data set: A reconcilation of the literature. *Journal of Manufacturing and Operations Management*, (2):199–221, 1998.

[7] J. L. Balintfy. On a basic class of multi-item inventory problems. *Management Science*, (10):287–297, 1964.

[8] M. Bastian. Joint replenishment in multi-item inventory systems. *Journal of the Operational Research Society*, (37):1113–1120, 1986.

[9] G. P. Cachon. Managing a retailers shelf space, inventory, and transportation. *MANUFACTURING and SERVICE OPERATIONS MANAGEMENT*, 3(3):211229, 2001.

[10] G. P. Cachon. *Supply Chain Management: Design, Coordination and Operation*, volume 11 of *Handbooks in Operations Research and Management Science*, chapter 6, Supply Chain Coordination with Contracts, pages 229–332. Elsevier, Amsterdam, Netherlands, 2003.

[11] H. Canaday. Oems in mro. *Overhaul and Maintenance*, VII(8), 2001.

[12] S. Cetinkaya and C. Lee. Stock replenishment and shipment scheduling for vendor managed inventory systems. *Management Science*, 46:217–232, 2000.

[13] A. K. Chakravarty. Multi-item inventory aggregation into groups. *Journal of the Operational Research Society*, (32):19–26, 1981.

[14] A. K. Chakravarty. An optimal heuristic for coordinated multi-item inventory replenishements. *Journal of the Operational Research Society*, (36):1027–1039, 1985.

[15] B. A. Chaouch. Stock levels and delivery rates in vendor-managed inventory programs. *Production and Operations Management*, 10(1):31–44, 2001.

[16] K. L. Cheung and H. L. Lee. The inventory benefit of shipment coordination and stock rebalancing in supply chain. *Management Science*, 48:300–306, 2002.

[17] K. Choi, J. G. Dai, and J. Song. On measuring supplier performance under vendor-managed-inventory programs in capacitated supply chains. *Manufacturing and Service Operations Management*, 6(1):53–72, 2004.

[18] M. Christopher. *Logistics and Supply Chain Management.* Financial Times/Prentice Hall, 1998.

[19] J. Corbett. Stochastic inventory systems in a supply chain with symmetric information: Cycle stocks, safety stocks and consignment stock. *Operations Research*, 49(4):487–500, 2001.

[20] J. Corbett, J. D. Blackburn, and L. N. V. Wassenhove. Case study: Partnerships to improve supply chains. *Sloan Management Review*, 1999.

[21] A. Federgruen, H. Groenevelt, and H. C. Tijms. Coordinated replenishments in a multi-item inventory system with compound poisson demand. *Management Science*, (30):344–357, 1984.

[22] M. J. Fry, R. Kapuscinski, and T. L. Olsen. Coordinating production and delivery under a (z,z)-type vendor-managed inventory contract. *Manufacturing and Service Operations Management*, 3(2):151–173, 2001.

[23] Y. Fu and R. Piplani. Supply side colloboration and its value in supply chains. *European Journal of Operational Research*, 152:281–288, 2004.

[24] S. K. Goyal. Determination of economic packaging frequency of items jointly replenished. *Management Science*, (20):232–235, 1934.

[25] S. K. Goyal. Determination of optimal packaging frequency of jointly replenished items. *Management Science*, (21):436–443, 1974.

[26] S. K. Goyal. Optimum ordering policy for a multi-item single supplier system. *Operational Research Quarterly*, (25):293–298, 1974.

[27] S. C. Graves. On the deterministic multi-product single-machine lot scheduling problem. *Management Science*, (25):276–280, 1979.

[28] G. Hadley and T. M. Whitin. *Analysis of Inventory Systems*. Prentice-Hall, NJ, 1963.

[29] F. Harris. Operations and cost. *Factory Management Series*, pages 48–52,, 1913.

[30] W. H. Hausman. Supplier managed availability. Web Report, www.supplychainonline.com, October 2003.

[31] E. Ignall. Optimal continuous review policies for two product inventory systems with joint set-up costs. *Management Science*, (15):278–283, 1969.

[32] D. M. Lambert and J. R. Stock. *Strategic Logistics Management*. Irwin, Homewood.

[33] J. Lee and L. Schwarz. Delegating inventory systems with agent-influenced replenishment leadtimes. Unpublished, 2004.

[34] V. Mahendroo. Service redefines supplier/customer relationship. *Semiconductor Magazine*, 1(1), 2000.

[35] P. Melchiors. Calculating can-order policies for the joint replenishment problem by the compensation approach. *European Journal of Operational Research*, (141):587–595, 2002.

[36] Pan Pro L.L.C. Vendor managed inventory primer. Web Report, www.supplychainonline.com, March 2004.

[37] P. Pantumsinchai. A comparison of three joint ordering inventory policies. *Decision Sciences*, (23):111–127, 1992.

[38] U. Persson. *A Conceptual and Empirical Examination of the Management Concept Supply Chain Management*. PhD thesis, Lule University of Technology, Sweden, 1997.

[39] R. Piplani and S. Viswanathan. A model for evaluating supplier-owned inventory strategy. *International Journal of Production Economics*, 81-82:565–571, 2003.

[40] B. Renberg and R. Planche. Un modèle pour la gestion simultanée des n articles d'un stock. *Revue Francaise d'Informatique et de Recherche Oprationelle*, Serie Blue:47–59, 1967.

[41] J. P. Schaack and E. A. Silver. A procedure, involving simulation for selecting the control variables of an (s,c,s) joint ordering strategy. *INFOR*, (10):154–170, 1972.

[42] H. Schultz and S. G. Johansen. Can-order polices for coordinated inventory replenishment with erlang distributed times between ordering. *European Journal of Operational Research*, (113):30–41, 1999.

[43] E. A. Silver. Some characteristics of a special joint-order inventory model. *Management Science*, (14):319–322, 1965.

[44] E. A. Silver. Three ways of obtaining the average cost expressions in a problem related to joint replenishment inventory control. *Naval Research Logistics Quarterly*, (20):241–254, 1973.

[45] E. A. Silver. A control system for co-ordinated inventory replenishment. *International Journal of Production Research*, (12):647–671, 1974.

[46] E. A. Silver. Establishing reorder points in the (s,c,s) co-ordinated control system under compound poisson demand. *International Journal of Production Research*, (19):743–750, 1981.

[47] E. A. Silver, D. F. Pyke, and R. Peterson. *Inventory Management and Production Planning and Scheduling.* John Wiley, New York, 1998.

[48] B. D. Sivazlian. A multi-commodity inventory system with setup costs. *Operations Research*, (7):213–240, 1970.

[49] S. Stremersch, S. Wuyts, and R. T. Frambach. The purchasing of full-service contracts: An exploratory study within the industrial maintenance market. *Industrial Marketing Management*, (30):1–12, 2001.

[50] R. M. Thompstone and E. A. Silver. A co-ordinated inventory control system for compound poisson demand and zero lead time. *International Journal of Production Research*, (13):581–602, 1975.

[51] A. A. Tsay, S. Nahmias, and A. N. *Quantitative Models for Supply Chain Management*, chapter 10, Quantitative Models for Supply Chain Management, pages 299–336. Kluwer, Boston, USA, 1999.

[52] G. Valentini and L. Zavanella. The consignment stock of inventories: industrial case and performance analysis. *International Journal of Production Economics*, 81-82:215–224, 2003.

[53] M. J. G. VanEijs. Analysis and comparison of 2 strategies for multiitem inventory systems with joint replenishment costs. *European Journal of Operational Research*, (59):405–412, 1992.

[54] S. Viswanathan. A new optimal algorithm for the joint replenishment problem. *Journal of the Operational Research Society*, (47):936–44, 1996.

[55] S. Viswanathan. On optimal algorithms for the joint replenishment problem. *Journal of the Operational Research Society*, (53):1286–1290, 2002.

[56] H. M. Wagner and T. M. Whitin. Dynamic version of the economic lot size model. *Management Science*, (5):89–96, 1958.

[57] Y. Wang, J. Li, and Z. Shen. Channel performance under consignment contract with revenue sharing. *Management Science*, 50(1):34–47, 2004.

[58] D. Waters. *Global Logistics and Distribution Planning*. Kogan Page, London, 2003.

[59] R. H. Wilson. A scientific routine for stock control. *Harvard Business Review*, (13):116–128, 1934.

[60] P. Zipkin. Inventory service-level measures: Convexity and approximation (working paper). *Graduate School of Business, Columbia University*, 1984.

[61] P. H. Zipkin. *Foundations of Inventory Management*. McGraw-Hill, Boston, 2000.