ORDER TIMING FOR SEASONAL PRODUCTS WITH DEMAND LEARNING AND CAPACITY CONSTRAINTS

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By
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August, 2009

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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Order time and order quantity of seasonal products significantly affect profits gained at the end of the period due to high demand uncertainty. Delaying order time enables a company to gain more information on demand, while decreasing the possibility of realizing the best order quantity due to capacity constraints. This thesis analyzes the problem of determining the best order time for a seasonal product manufacturer in an environment, where there exists a single opportunity for ordering and capacity is a decreasing function of the order time. Main feature of the study is utilizing demand information collected until the order time for resolving some portion of the demand uncertainty. A Bayesian update procedure is utilized to capture the essence of the gathered demand information. Three models are proposed for determining the order time, each having a different level of flexibility with respect to possible order times considered. Analytical results for structural properties, as well as extensive numerical results are obtained. A computational study is carried out in order to compare the performance of the models under different settings and to identify the conditions under which the demand learning is most beneficial.

Keywords: seasonal products inventory problem, order time, Bayesian information updating.

ÖZET

SEZONSAL ÜRÜNLER İÇİN TALEP BİLGİSİ GÜNCELLEME VE KAPASİTE KISITI ALTINDA SİPARİŞ ZAMANLAMASI

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Ağustos, 2009

Talep belirsizliği yüksek olan sezonsal ürünler için verilen siparişin zamanı ve miktarı dönem sonunda elde edilecek karlılığı önemli ölçüde etkiler. Sipariş zamanını geciktirmek, bir yanda, belirsiz talep hakknda daha fazla bilgi edinilmesine imkan verirken, öte yanda, kapasite kısıtlarından dolayı istenen ideal sipariş miktarının üretilme olasılığını azaltabilir. Bu çalışmada, tek sezonda satılan bir ürün ve üretim için kullanılabilecek kapasitenin dönem sonuna kadar olan zamanının doğrusal bir fonksiyonu olduğu bir ortamda en iyi sipariş zamanını belirleme problemi incelenmiştir. Yapılan çalmanın ana öğesi, sipariş zamanına kadar verilen müşteri siparişlerini gözlemleyerek talep dağılımının parametrelerinin Bayes yaklaşımı ile güncellenebilmesidir. Sipariş zamanının statik olarak belirlenebildiği iki model ve dinamik olarak belirlenebildiği bir model geliştirilmiş, bu kararın verilmesine ışık tutacak analitik ve sayısal sonuçlar elde edilmiştir.

Anahtar sözcükler: sezonsal ürünler, sipariş zamanı, Bayes tipi güncelleme.

To my parents...

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Chapter 1

Introduction

Main features of seasonal products (e.g. style goods) are short selling season with a definite beginning and end, demand uncertainty arising from both long period of inactivity between seasons and introduction of newly designed products in each season, long lead time between order and delivery, and commitment of order amounts prior to the selling season [19]. In order to resolve demand uncertainty, sales data of similar products sold in previous years or expert opinions are used. However, most of the uncertainty still remains because of ever changing consumer tastes and varying economic conditions. Due to high setup costs and other economies of scale, usually one setup is made for production of this type of products. Facing with long lead times, high setup cost and short selling season constrain order times for both manufacturers and retailers. Generally, order decisions are taken prior to the season, before any demand is realized. Therefore, matching supply and demand becomes increasingly difficult, which results in either excess inventory leading to high inventory carrying costs and high markdown costs or stockouts leading to high stockout costs and low service levels. Frazier [10] estimates the profit losses in U.S. apparel industry as $25 billion due to excess supply and shortages. The rapid developments in technology and innovations increase variety of products, which increase the difficulty of matching supply and demand tremendously. In department stores markdowns increase from 8% to 26% of sales between 1972 and 1990 [8].

A classical example of mismatch between supply and demand is the case of Sport Obermeyer [9, 8, 7]. Sport Obermeyer is a major supplier of U.S. fashion ski-wear industry that both designs and manufactures ski apparel products. As it is known, in this industry it is hard to forecast the demand due to fashion trends, weather conditions, economic conditions and newly designed products each year. The retail selling season is between September 1 and December for urban stores and September 1 and mid-February for ski-area stores. The products are manufactured in different countries like Hong Kong, China, Japan, Korea, Jamaica, Bangladesh, and United States. The production starts on January 1, nearly eight months in advance of the season, and ends on September 1. The suppliers produce based on production orders of Obermeyer. The samples of the products are shown to retailers in February, they order between mid-February and May, and the orders are delivered to them by October. Reorders between October and December, which is approximately 10% of sales, are satisfied from available inventory and after January 1 remaining inventory is sold with markdowns. Since manufacturer starts production without observing any demand and finishes before realizing all of the demand, the risk of mismatched supply and demand is very high and determining production order quantities is a challenging task. For example; in 91/92 season, sales of a group of women parkas were 200% higher than the forecasted value, whereas sales were 15% lower than the forecasted value for another group.

By the initiative of U.S. apparel industry, a strategy called Quick Response (QR) is developed in response to inflexible production environment and uncertain demand of style goods [13]. QR focuses on shortening lead times through developments in various operations like manufacturing methods, information and communication technologies, and logistics. As a result, order or production decisions can be made closer to the selling season or in the initial part of the season. An essential benefit gained from QR is that forecasts can be adjusted by utilizing early sales information collected from the market, which reduces the forecast errors and consequently inventory and stockout costs.

As mentioned below, a strategy for resolving demand uncertainty is gathering information from the market for a certain amount of time and improving the quality of the forecasts based on market signals. In this case, production or order amounts are determined based on the new estimate of demand, which reduces the mismatches between supply and demand. Fisher and Raman [9] illustrate the enormous improvement in forecasts by comparing initial forecasts and updated forecasts based on the first 20% of demand. While deciding on the duration of demand observation (order time), capacity constraint should also be considered. A key point that should be taken into account is that delaying production order time on one hand provides more accurate demand information; on the other hand reduces the possibility of realizing optimal order quantity due to capacity restrictions.

One of the widely used approaches for updating demand forecasts by incorporating observations is Bayesian approach. It is generally assumed that demand is distributed with a known parameter (or parameters), however this is not always the case. Especially for the style goods, whose demand uncertainty is high due to inconsiderable demand history, it is hard to estimate the true value of the distribution parameters. For this reason, it is assumed that demand through the season is random with a specified distribution, whose parameter is not known. A prior distribution is assigned to the unknown parameter, which denotes the initial estimate or beliefs on demand. This distribution is updated as new information on demand becomes available. As new information becomes available, the distribution is improved continuously so that the demand can be represented with its true distribution. For efficient use of Bayesian approach, there exist conjugate prior distributions corresponding to a specified demand distribution, which can be used for the unknown parameter. It is obvious that different choices from the conjugate priors will increase the difficulty of the update procedure. However, the use of the conjugate priors makes the demand learning a dynamic process, in which parameters of posterior distribution change with information over time. A few examples are as follows. Gamma is conjugate for Normal, Exponential and Poisson distributions, Beta is conjugate for Geometric, Binomial, Negative Binomial and Bernoulli distributions etc. (See [11] for more examples). The only disadvantage of this approach is that the formulation of a prior distribution for the unknown parameter is a very hard task when the decision maker has absolutely no idea about the unknown parameter. Fortunately, this is not valid for our problem.

For our problem we consider environments similar to Sport Obermeyer's case. There is a manufacturer that supplies a seasonal product to several retailers. There is a well defined period before the retail selling season, in which the manufacturer places order once to its supplier. The end of this period is at least as early as the lead time, which is required for manufacturing and distribution of the product, so that the products reach to the retailers on time. Hence, the manufacturer has to determine the order time and quantity carefully. Note that retailers can place orders to the manufacturer based on catalogs sent or samples shown through this ordering period; by this way manufacturer can observe the demand.

This thesis analyzes the problem of determining the best order time and corresponding order quantity for the aforementioned environment by utilizing demand learning. By demand learning, the demand information collected until the order time is used for increasing the quality of the demand estimate. Note that the problem under consideration focuses on environments in which there exists a single opportunity for ordering and no further opportunities for adjusting the order quantity. We assume that the supplier has linearly decreasing capacity with respect to time, but nonlinear decreasing structure can also be analyzed with a similar fashion. We use a specific form of Bayesian approach for our demand model, which assumes that the demand is distributed by Poisson with an unknown parameter. The unknown parameter's prior distribution is assumed to be Gamma, which produces Negative Binomial distribution for the unconditional distribution of demand. (These standard distributions are also used in Sen and Zhang's [5] study on style goods pricing.) One dynamic and two static models are developed for choosing the best order time under these assumptions. The first model chooses the order time from two predetermined order times, while the second model finds the best order time depending on the observed demand until a predetermined time. Contrary to these models, the third model is a dynamic one which evaluates each time point as a possible order time. Note that the models focus on the trade-off between capacity and demand learning. Analytical and numerical results are derived in order to understand the behavior of the best order time. We also carry out computational studies in order to compare different models under different settings and it is observed that Model 3 outperforms the other models for the majority of the cases considered. Lastly, the value of demand learning is assessed by changing the true value of the Poisson rate and the variance of the initial point estimate. The details of this study can be found in Chapter 5.

The rest of this thesis is organized as follows. In Chapter 2, related literature is summarized. In Chapter 3, demand model and models developed for specification of the best order time are presented. In Chapter 4, analytical and numerical results are derived to understand how each model operates. In Chapter 5 computational studies are carried out under different settings for comparison of different models and the conditions under which demand learning is most beneficial are highlighted. Finally, the thesis is concluded with a summary of results and possible extensions in Chapter 6.

Chapter 2

Literature Review

Inventory management is a key issue faced by managers dealing with seasonal products. As a consequence of rapid developments and innovations in the technology, and globalization, product life cycles shorten tremendously and matching supply with demand becomes a major challenge. Demand learning is a current solution for resolving some portion of demand uncertainty. By demand learning, we imply the revision of forecasts based on early sales information. In this chapter, we present a brief review of studies concerned with inventory management of seasonal products and demand learning.

The stochastic single period inventory model is known as Newsboy Model and dates back to 1950s. The classical Newsboy Model assumes that orders are committed once at the beginning of the season, backorders are fully backlogged and inventory surplus is not transferred to the next season [19]. It focuses on finding order quantity that minimizes the expected cost or maximizes the expected profit. An extensive literature deals with newsboy type problems and various extensions for the classical model have been suggested. Khouja [14] presents an intense discussion of extensions suggested for single period problem and provides a taxonomy of the literature so far.

Inventory models including demand learning has received considerable attention in the literature. Scarf [20] is the first author that incorporates demand learning in an inventory modeling context. He develops a dynamic inventory model that uses observed demand information and current stock level together in the decision process. He assumes that demand is generated from exponential class of distributions and a conjugate prior distribution is used for the unknown parameter. The distribution is updated by Bayesian approach at the beginning of each period. Scarf [21] and Azoury [1] are other two examples of early studies that incorporate demand learning in dynamic inventory systems.

Inventory models of seasonal products incorporating demand learning have been extensively studied. Demand learning is crucial for this type of products due to inflexible production environment and highly uncertain nature of the products. The models have considered different scenarios, but most of them use Bayesian approach for adjustment of demand distribution parameters by utilizing early sales information. Murray and Silver [18] present one of the earliest work that considers demand learning for inventory modeling of style goods. They assume that there are known number of customers, but buying potential of each customer is stochastic. Beta distribution is used for prior distribution of purchase probabilities. At each acquisition time, the purchase probabilities are updated based on observed sales and optimal order quantity is decided considering both stock on hand and sales information.

Iyer and Bergen [13] analyze the effects of QR on manufacturer-retailer channel by using newsboy type inventory models that incorporates demand learning through Bayesian approach. They assume that the demand includes two sources of uncertainty; uncertainty due to product and mean demand uncertainty at the beginning of the season. They use a particular form of Bayesian approach that assumes Normal distribution for both demand and unknown parameter. Eppen and Iyer [6] present a special form of quantity flexibility contracts, backup agreements between a catalog company and manufacturer for a two period setting. They introduce a different version of Bayesian approach, in which it is assumed that demand through the season is from a set of pure demand processes and the true demand process is unknown at the beginning of the season. Prior probabilities are assigned to demand processes and these are updated by Bayes' rule as demand information becomes available. The approach restricts the distributions that can be used for demand processes; some appropriate distributions are Normal, Negative Binomial and Poisson.

There are also alternative methods used for updating forecasts in the literature. Chang and Fyffe [3] propose a methodology for revising forecasts based on early sales considering a season with multiple periods. They assume that demand in each period is a fixed fraction of the aggregate demand plus a noise term. Total demand distribution is updated as new demand information becomes available. Hausman [12] shows that successive demand forecasts are independent random variables and distributed by lognormal distribution, under certain conditions. He assumes that demand shows markovian property; demand in each period is related to past demand only through the demand in the previous period.

There is also substantial amount of research concerned with determining order quantities of seasonal products at two order opportunities by including demand learning. The studies on this issue show difference in terms of scenarios on order times, order or production cost and supplier capacity. Fisher and Raman [9] model a fashion goods production environment as a two stage stochastic program and solve it using Lagrangian relaxation method. One of the main assumption is that production decisions are taken at predetermined two distinct time points. Firstly, an initial production order is given before observing any demand, then a second order is placed based on updated forecasts. The model focuses on finding distinct production amounts at two points subject to minimum lot size quantities and second period's production capacity. Also, the paper presents a method for the estimation of demand probability distributions that combines historical data of similar products and expert opinions. A key feature of demand distributions is that it allows correlation between demand of first and second period; in particular the total and first period's demand is assumed to be Bivariate Normal. Choi et al. [4] also derive an optimal two stage ordering policy for a single seasonal product by utilizing dynamic programming and Bayesian information updating. The demand model is similar to one used in Iyer and Bergen [13]. At the first stage, ordering cost is known and the demand forecast uncertainty is high whereas at the second stage ordering cost is uncertain and the demand forecast uncertainty is lower. They also study the performance of optimal ordering policy in terms of service level and variance of profit and present further numerical analysis to show performance of the policy under different parameter settings. One of the recent works on this issue is Miltenburg and Pong [16] that also uses Bayesian information updating. They consider two order opportunities; one with low ordering cost and one with higher ordering cost. The total order quantity calculated at first order time is adjusted at the second order opportunity based on updated demand distribution. They also examine some standard distributions used for Bayesian procedure by providing examples for each. Miltenburg and Pong [17] extends this study by including capacity restriction at each order opportunity.

All of these studies ignore one crucial aspect: the impact of the order time. Fisher et al. [8] analyze the impact of several operational changes on the profit of the system including order time, verbally. They perform a simulation study with the data of Sport Obermeyer in order to quantify effects of the operational parameters like production capacity, minimum production lot size and lead time on the expected cost. The company under investigation gives production orders at two distinct time points and the model in Fisher and Raman [9] is used for finding allocation of production quantities of items between two points and its associated expected costs. Their results indicate that the expected stockout and markdown costs decrease with the increase in the percentage of demand observed, as long as there exists considerable capacity.

All of these studies discussed above ignore the determination of the order time, which may has a significant impact on the end of period profits. Main focus of our study is the determination of the best order time and the corresponding order quantity by minimizing total expected costs subject to remaining capacity. Our main contribution to the literature is that we present static and dynamic models that incorporate Bayesian type demand learning and illustrate the trade-off between more accurate demand information and decreasing capacity.

Chapter 3

Model Formulation

In this chapter, we first introduce the demand model. Then, basic definitions and assumptions of the problem are briefly explained. This is followed by the description of static and dynamic models that we develop to solve order timing problem by incorporating demand learning.

3.1 Demand Model

In this section, we explain our demand model and show how demand learning is used to update the parameters of demand distribution. Note that the derivation of the prior and posterior distributions can be found in Chapter 4 in [2].

We assume, without loss of generality, that the ordering period is of unit length. Let there be $T-1$ possible order times during the period, denoted by t_1, t_2, $\cdots$, t_{T-1}. Also, denote $t_0 = 0$ and $t_T = 1$. Denote X_{ij} to be the demand between t_i and t_j (with X_{0T} corresponding to the total demand during the period). An example with $T = 3$ is given in Figure 3.1.

Assume that the demand during the whole period (X_{0T}) is distributed with Poisson with an unknown parameter Λ, i.e.,

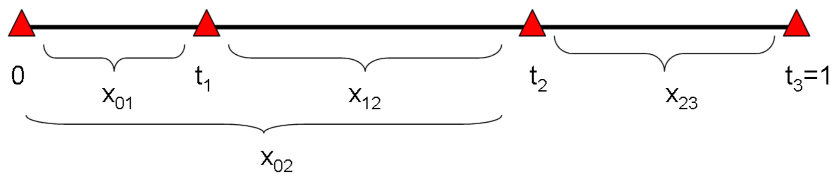

Figure 3.1: Time Line of Ordering Period for Model 1

$$p(x_{0T}) = \tfrac{e^{-\Lambda}\Lambda^{x_{0T}}}{x_{0T}!}, \text{ for } x_{0T} = 0, 1, 2, \cdots$$

This leads to demand in each interval $(t_i,\ t_j]$ to be distributed by Poisson with parameter $\Lambda(t_j - t_i)$.

Assume that Λ's prior distribution is Gamma with parameters α and β, i.e.,

$$f(\lambda) = \tfrac{\beta^\alpha \lambda^{\alpha-1} e^{-\beta\lambda}}{\Gamma(\alpha)}, \ \lambda > 0$$

Then, the prior distribution of total demand unconditional of Λ can be found as follows:

$$\begin{aligned}
p(x_{0T}) &= \int_0^\infty p(x_{0T}|\Lambda = \lambda) f(\lambda) d\lambda \\
&= \int_0^\infty \frac{e^{-\lambda}\lambda^{x_{0T}}}{x_{0T}!} \frac{\beta^\alpha \lambda^{\alpha-1} e^{-\beta\lambda}}{\Gamma(\alpha)} d\lambda \\
&= \frac{\beta^\alpha}{x_{0T}!\Gamma(\alpha)} \int_0^\infty \lambda^{(x_{0T}+\alpha)-1} e^{-\lambda(1+\beta)} d\lambda \\
&= \frac{\beta^\alpha}{x_{0T}!\Gamma(\alpha)} \frac{\Gamma(x_{0T}+\alpha)}{(1+\beta)^{x_{0T}+\alpha}} \\
&= \frac{\Gamma(x_{0T}+\alpha)}{\Gamma(\alpha) x_{0T}!} \left(\frac{\beta}{\beta+1}\right)^\alpha \left(\frac{1}{\beta+1}\right)^{x_{0T}} \\
&= \binom{x_{0T}+\alpha-1}{x_{0T}} \left(\frac{\beta}{\beta+1}\right)^\alpha \left(\frac{1}{\beta+1}\right)^{x_{0T}}
\end{aligned}$$

which is the distribution function of a Negative Binomial random variable with parameters α and $\beta/(\beta+1)$. Thus, we write,

$$X_{0T} \sim NB\left(\alpha, \frac{\beta}{\beta+1}\right). \tag{3.1}$$

Similarly, unconditional distribution of the demand in interval $(t_i, t_j]$ can be found to be (prior to any observation) Negative Binomial with parameters α and $\beta/(\beta+t_j-t_i)$:

$$p(x_{ij}) = \binom{x_{ij}+\alpha-1}{x_{ij}} \left(\frac{\beta}{\beta+t_j-t_i}\right)^{\alpha} \left(\frac{t_j-t_i}{\beta+t_j-t_i}\right)^{x_{ij}}, \quad \text{or}$$

$$X_{ij} \sim NB\left(\alpha, \frac{\beta}{\beta+t_j-t_i}\right). \tag{3.2}$$

If the realized demand in an interval $(t_i, t_j]$ is x_{ij}, then the posterior distribution of the demand rate can be derived by Bayes' rule as follows:

$$\begin{aligned} f(\lambda|X_{ij}=x_{ij}) &= \frac{f(\lambda)p(x_{ij}|\Lambda=\lambda)}{\int_0^\infty f(\lambda)p(x_{ij}|\Lambda=\lambda)d\lambda} \\ &= \frac{\lambda^{\alpha+x_{ij}-1}(\beta+t_j-t_i)^{\alpha+x_{ij}}e^{-(\beta+t_j-t_i)\lambda}}{\Gamma(\alpha+x_{ij})} \end{aligned}$$

After evaluating the integral and some simplification, the posterior distribution of Λ can be shown to be Gamma with shape parameter $(\alpha+x_{ij})$ and scale parameter $(\beta+t_j-t_i)$.

Therefore, the posterior distribution of a future interval $(t_k, t_l]$ will be distributed with Negative Binomial distribution as follows:

$$(X_{kl}|X_{ij}=x_{ij}) \sim NB\left(\alpha+x_{ij}, \frac{\beta+t_j-t_i}{\beta+t_j-t_i+t_l-t_k}\right). \tag{3.3}$$

As anticipated, the expected value of demand in a future period $(t_k, t_l]$ linearly depends on observed sales in period $(t_i, t_j]$ as follows:

$$E[X_{kl}|X_{ij} = x_{ij}] = \frac{(\alpha+x_{ij})(t_l-t_k)}{\beta+t_j-ti}.$$

We know that utilizing Bayesian approach brings convergence of posterior distribution of demand to its underlying true distribution as length of the period approaches to infinity. Then, as time passes and more information is accumulated posterior distribution of total demand approaches to the true distribution of total demand. Thus, most of the demand uncertainty is resolved and the possibility of mismatch between supply and demand decreases. The general line of the argument can be followed by Figure 3.2, over an example with period length of 1. The figure includes cumulative distribution function of total demand for the true distribution, prior distribution and posterior distribution at three time points with different observations. For the example, we assume that underlying true distribution of demand is Poisson with $\Lambda = 20$ and the prior distribution of Λ is Gamma with parameters 15 and 0.5. We choose the distribution parameters, time points and associated observed demand values arbitrarily so that this argument can be seen clearly. For this particular case, as time passes and more observations are collected the posterior distribution progressively approaches to the true distribution starting from the prior distribution. Note that for this case, the convergence may not always be observed, since the period of finite length.

3.2 Assumptions and Problem Definition

There is a well-defined ordering period for manufacturers before the retail selling season, which is assumed to be of unit length and includes the potential order times. The end of the ordering period is defined, so that it is at least lead time (of manufacturing and distributing products) earlier than the start of the selling season. The manufacturer has a single opportunity to order from its suppliers throughout the period. Starting from the start of the period, the demand from retailers is observed until each given possible order time point and these observations are used to update the forecasts for the remainder of the period. Costs under consideration are unit purchase, inventory holding and shortage costs. Note that the problem is modeled using the notation given in Table 3.1 and additional

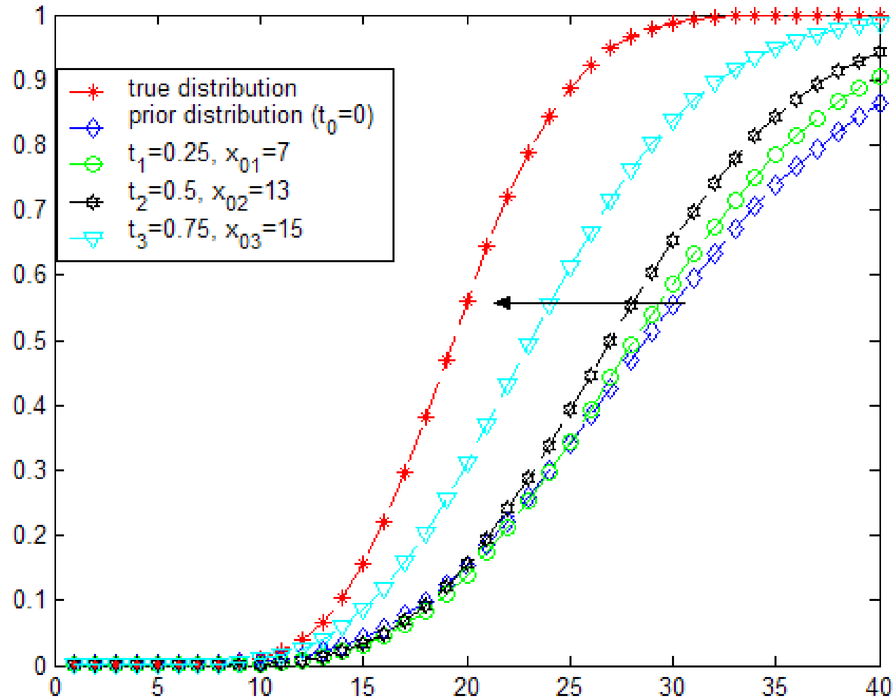

Figure 3.2: An example for Bayesian approach with Λ=20, α=10, $\beta = 0.5$

notation is defined and explained as needed. If total demand in the period turns out to be higher than the order quantity, for every unit short a shortage cost b is charged to the manufacturer at the end of the period. If total demand is less than the order quantity, inventory holding cost h is charged per unit at the end of the period. Fixed costs of ordering (or setup) and the cost of observing demand are neglected. A total capacity of c units is assigned for the whole period and it is assumed that the capacity is linearly decreasing over time.

The problem under consideration is to determine the best order time and the corresponding order quantity given that capacity is decreasing and more demand information is gathered as time progresses. The primary trade-off in this problem is the trade-off between additional demand information and remaining capacity. We prepare three different models for this problem. The first model chooses the best order time from two possible order times t_1 and t_2, at t_1 that are defined well in advance of the ordering period. This could be the case, when supplier has

m :	Unit purchasing cost
b :	Unit shortage cost, $b > m$
h :	Unit inventory holding cost
c :	Total capacity through the period
X_{ij} :	Demand between t_i and t_j (a r.v.)
$p(x_{ij})$:	Probability mass function of X_{ij}
$P(x_{ij})$:	Cumulative distribution function of X_{ij}
$\mu_{x_{ij}\|x_{kl}}$:	Expected value of X_{ij} given $X_{kl} = x_{kl}$
$C(t_i)$:	Remaining capacity at t_i, $C(t_i) = (1 - t_i)c$
y_i :	Order quantity at i^{th} order time
$\overline{y}_i$:	Optimal order quantity at i^{th} order time
${y_i}^*$:	Optimal order quantity at i^{th} order time under capacity restriction
$EC_{t_i,x_{0i}}[y_i]$:	Expected total cost of ordering y_i units at t_i given that total observed demand until t_i is x_{0i}
$EC^*_{t_i,x_{0i}}$:	Optimal expected total cost of ordering at t_i given that total observed demand until t_i is x_{0i}

Table 3.1: Notation

more power than the manufacturer and so controls the order time by dictating two possible order times before the ordering period. He forces the manufacturer to decide at t_1 whether to place the order at this instant or delay to t_2. The second model finds the best order time based on the observed demand until predetermined first possible order time t_1. For this case, the supplier provides more flexibility regarding the order time. He announces the first possible order time and allow the manufacturer to determine and declare the order time based on the observations until t_1. The last model is a dynamic one, which considers each time point as a possible order time and decides to order or continue observing demand based on the total observed demand until that point. This could be the case, when the manufacturer has more control over the order time and can place the order whenever he wants. For all models, the objective function is the expected total cost including purchasing, inventory and shortage costs. The detailed discussion of the models are presented in Sections 3.3, 3.4 and 3.5.

3.3 Model 1

For this model it is assumed that there are two fixed order times at t_1 and t_2 that are specified prior to the ordering period. The period is divided into three subperiods which are defined as $[0, t_1]$, $(t_1, t_2]$ and $(t_2, 1]$ and the demand observed in them are x_{01}, x_{12} and x_{23}, respectively as seen from Figure 3.1. The problem is to choose the order time, from these two predetermined times, which minimizes the expected cost of the decision maker. t_1 is the earliest time that the manufacturer can order. Until t_1, manufacturer collects information from the market and updates the demand distribution accordingly by utilizing demand model described in Section 3.1. At this point retailer has to decide whether to order at this instant or delay it until t_2 by comparing the expected cost of ordering at t_1 and t_2. Note that the order quantities and its associated expected costs are calculated by using updated distribution of demand. After comparison, the time point with smaller expected cost is chosen as the order time. If retailer chooses to wait until t_2, then the demand distribution is again updated with respect to realized demand in the second subperiod and the order quantity is decided based on the new distribution. While deciding on the order time, the remaining capacity at each order time is also considered. Delaying order time from t_1 to t_2 allows the decision maker to learn more about the demand, hence makes the order quantity to better respond to demand. On the other hand, it decreases the possibility of ordering the desired amount due to capacity constraints. In other words, at t_2 we have more accurate demand information at the risk of insufficient capacity.

Observation 0: When we evaluate the expected cost of ordering at t_1 with small grid size between 0 and 1, we see that it is decreasing until a certain point and then it shows sawtooth structure between two capacity change points. In Figure 3.3, we illustrate an example for this. The reason behind this is that remaining capacity for such a time point and the subsequent point is the same. To be more explicit, since the demand is discrete a capacity of 34.99 and 34 is the same. However the uncertainty decreases by the information obtained over time and this makes the value of delaying the order decision always positive. Therefore, the order times should be chosen from the time points for which a capacity change occurs.

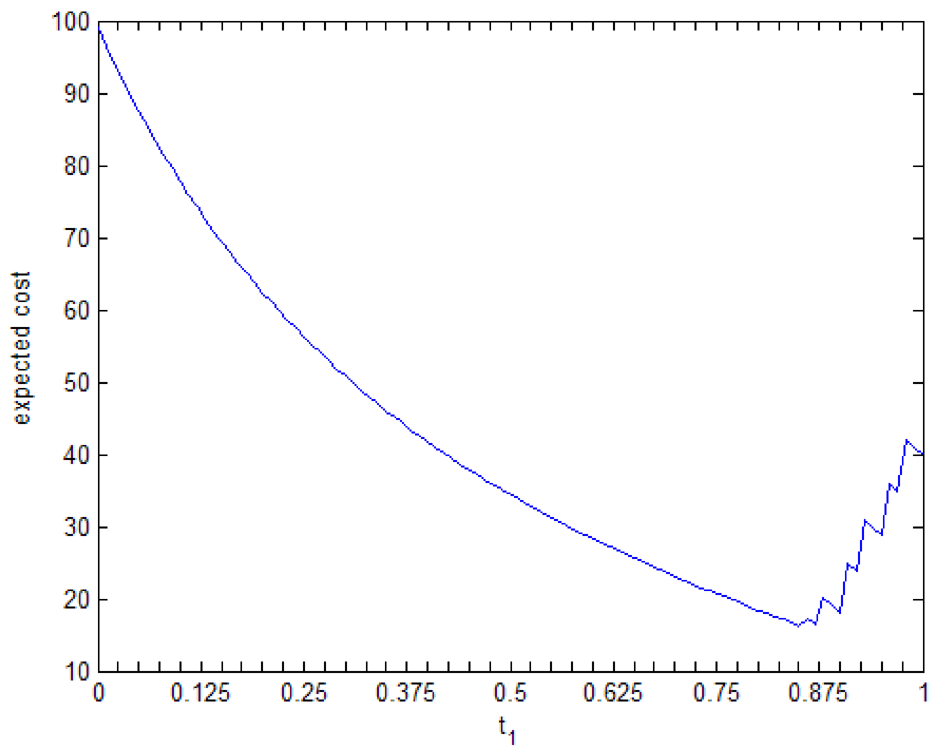

Figure 3.3: An example for *Observation* 0 with m=2, b=10, h=1, c=40, x_{01}=4, α=10, $\beta = 0.5$

In accordance with this observation, throughout the study we choose the possible order times from points where capacity changes. Hence, optimization is carried out over discrete time points. Note that in our models, the possible order times under consideration are multiples of $1/c$ due to remaining capacity structure.

The decision process of the model is summarized in four steps.

Step 1: Given that order decision is made at t_1 and x_{01} is observed so far, find the best order quantity and the corresponding expected cost based on the updated demand distribution with x_{01}.

$$y_1^* = \min_{0 \le y_1 \le C(t_1)} \quad EC_{t_1,x_{01}}[y_1]$$

$$\begin{aligned} \text{where} \quad EC_{t_1,x_{01}}[y_1] &= my_1 + \sum_{x_{13}=0}^{y_1-x_{01}-1} h(y_1 - x_{01} - x_{13})p(x_{13}|x_{01}) \\ &+ \sum_{x_{13}=y_1-x_{01}}^{\infty} b(x_{13} - (y_1 - x_{01}))p(x_{13}|x_{01}) \end{aligned}$$

Note that $p(x_{ij}|X_{kl} = x_{kl})$ is denoted by $p(x_{ij}|x_{kl})$ and $P(x_{ij}|X_{kl} = x_{kl})$ is denoted by $P(x_{ij}|x_{kl})$ in the formulation of the models.

The expected cost can be simplified as follows:

$$\begin{aligned} EC_{t_1,x_{01}}[y_1] &= my_1 + \sum_{x_{13}=0}^{y_1-x_{01}-1} h(y_1 - x_{01} - x_{13})p(x_{13}|x_{01}) \\ &+ \sum_{x_{13}=y_1-x_1}^{\infty} b(x_{13} - (y_1 - x_{01}))p(x_{13}|x_{01}) \\ &+ \sum_{x_{13}=0}^{y_1-x_{01}-1} b(x_{13} - (y_1 - x_{01}))p(x_{13}|x_{01}) \\ &- \sum_{x_{13}=0}^{y_1-x_{01}-1} b(x_{13} - (y_1 - x_{01}))p(x_{13}|x_{01}) \\ &= my_1 + \sum_{x_{13}=0}^{y_1-x_{01}-1} (b+h)(y_1 - x_{01} - x_{13})p(x_{13}|x_{01}) \\ &+ \sum_{x_{13}=0}^{\infty} b(x_{13} - y_1 + x_{01})p(x_{13}|x_{01}) \\ &= (m-b)y_1 + b(x_{01} + \mu_{x_{13}|x_{01}}) \\ &+ (b+h) \sum_{x_{13}=0}^{y_1-x_{01}-1} (y_1 - x_{01} - x_{13})p(x_{13}|x_{01}). \end{aligned}$$

Let

$$
\begin{aligned}
\Delta EC_{t_1,x_{01}}[y_1] &= EC_{t_1,x_{01}}[y_1+1] - EC_{t_1,x_{01}}[y_1] \\
&= (m-b)(y_1+1) + b(x_{01} + \mu_{x_{13}|x_{01}}) \\
&+ (b+h)\sum_{x_{13}=0}^{y_1-x_{01}} (y_1 + 1 - x_{01} - x_{13})p(x_{13}|x_{01}) \\
&- (m-b)y_1 - b(x_{01} + \mu_{x_{13}|x_{01}}) \\
&- (b+h)\sum_{x_{13}=0}^{y_1-x_{01}-1} (y_1 - x_{01} - x_{13})p(x_{13}|x_{01}) \\
&= (m-b) + (b+h)P(y_1 - x_{01}|x_{01}).
\end{aligned}
$$

$\Delta EC_{t_1,x_{01}}[y_1]$ is the change in the expected total cost, when we switch from order quantity of y_1 to y_1+1. Since the cost function is discrete convex [19], the smallest y_1 which makes this value greater than zero will give the optimal y_1. Then, the decision rule is to select the smallest y_1 value ($\overline{y}_1$) that satisfies:

$$
P(y_1 - x_{01}|x_{01}) \geq \frac{b-m}{b+h}. \tag{3.4}
$$

The optimal order quantity at t_1 under capacity restriction and the optimal expected cost are:

$$
y_1^* = \begin{cases} \overline{y}_1 & \text{if } \overline{y}_1 \leq C(t_1), \\ C(t_1) & \text{otherwise.} \end{cases} \tag{3.5}
$$

$$
EC^*_{t_1,x_{01}} = \begin{cases} (m-b)y_1^* + b(x_{01} + \mu_{x_{13}|x_{01}}) & \text{if } x_{01} \leq C(t_1), \\ +(b+h)\sum_{x_{13}=0}^{y_1^*-x_{01}-1}(y_1^* - x_{01} - x_{13})p(x_{13}|x_{01}) & \\ & \\ (m-b)C(t_1) + b(x_{01} + \mu_{x_{13}|x_{01}}) & \text{if } x_{01} > C(t_1). \end{cases} \tag{3.6}
$$

Step 2: Given that order decision is made at t_2 and x_{02} is observed so far, find the optimal order quantity and corresponding expected cost based on updated demand distribution with x_{02}.

$$y_2^{*}{=}\min_{0\leq y_2\leq C(t_2)} \quad EC_{t_2,x_{02}}[y_2]$$

By similar reasoning with the previous problem, optimal order quantity $\overline{y}_2$ is the smallest value of y_2 that satisfies:

$$P(y_2 - x_{02}|x_{02}) \geq \frac{b-m}{b+h}. \tag{3.7}$$

The optimal order quantity at t_2 under capacity restriction and the optimal expected cost are:

$$y_2^{*} = \begin{cases} \overline{y}_2 & \text{if } \overline{y}_2 \leq C(t_2), \\ C(t_2) & \text{otherwise.} \end{cases} \tag{3.8}$$

$$EC^{*}_{t_2,x_{02}} = \begin{cases} (m-b)y_2^{*} + b(x_{02} + \mu_{x_{23}|x_{02}}) & \text{if } x_{02} \leq C(t_2), \\ +(b+h)\sum_{x_{23}=0}^{y_2^{*}-x_{02}-1}(y_2^{*} - x_{02} - x_{23})p(x_{23}|x_{02}) & \\ & \\ (m-b)C(t_2) + b(x_{02} + \mu_{x_{23}|x_{02}}) & \text{if } x_{02} > C(t_2). \end{cases} \tag{3.9}$$

Step 3: Given that x_{01} is observed at t_1, find the expected cost of ordering at t_2. To do this, take the expectation of $EC^{*}_{t_2,x_{01}+X_{12}}$.

$$E[EC^{*}_{t_2,x_{01}+X_{12}}]{=}\sum_{x_{12}=0}^{\infty} p(x_{12}|x_{01})EC^{*}_{t_2,x_{01}+x_{12}}$$

While calculating $E[EC^{*}_{t_2,x_{01}+X_{12}}]$, we should consider that the distribution parameters of last subperiod's demand changes with different values x_{12} which consequently produces different y_2^{*}s. In order to find a compact form of $E[EC^{*}_{t_2,x_{01}+X_{12}}]$, we need to find the largest x_{12} value which produces optimal order quantity without being subject to capacity constraint. This value is denoted by $\overline{x}_{12}$ and found by selecting the largest x_{12} satisfying the following inequality:

$$P(C(t_2) - x_{01} - x_{12}|x_{01} + x_{12}) \geq \frac{b-m}{b+h}. \tag{3.10}$$

At t_1, the optimal expected total cost of ordering at t_2 is;

$$\begin{aligned}
& E[EC^*_{t_2,x_{01}+X_{12}}] = \sum_{x_{12}=0}^{\infty} p(x_{12}|x_{01}) EC^*_{t_2,x_{01}+x_{12}} \\
= & \sum_{x_{12}=0}^{\overline{x}_{12}} [(m-b)y_2^* + b(x_{02} + \mu_{x_{23}|x_{02}}) \\
+ & (b+h) \sum_{x_{23}=0}^{y_2^*-x_{02}-1} (y_2^* - x_{02} - x_{23}) p(x_{23}|x_{02})] p(x_{12}|x_{01}) \\
+ & \sum_{x_{12}=\overline{x}_{12}+1}^{C(t_2)-x_{01}} [(m-b)C(t_2) + b(x_{02} + \mu_{x_{23}|x_{02}}) \\
+ & (b+h) \sum_{x_{23}=0}^{C(t_2)-x_{02}-1} (C(t_2) - x_{02} - x_{23}) p(x_{23}|x_{02})] p(x_{12}|x_{01}) \\
+ & \sum_{x_{12}=C(t_2)-x_{01}+1}^{\infty} [(m-b)C(t_2) + b(x_{02} + \mu_{x_{23}|x_{02}})] p(x_{12}|x_{01})
\end{aligned} \tag{3.11}$$

Step 4: Compare $EC^*_{t_1,x_{01}}$ and $E[EC^*_{t_2,x_{01}+X_{12}}]$ and choose the time with the smaller expected cost as the order time.

We now state an important property of the order decision at t_1.

Property 1: Optimal order quantity at t_1 is a non-decreasing function of the observed demand.

Proof. Let P_1 be the cdf of the remaining demand at t_1 when the observed demand is x_{01} and P_2 be the cdf of the remaining demand at t_1 when the observed demand is $x_{01}+1$. Let (r_1, p) and (r_2, p) be the Negative Binomial distribution's parameters for P_1 and P_2 respectively, where $r_1 = \alpha + x_{01}$, $r_2 = \alpha + x_{01} + 1$ and $p = (\beta + t_1)/(\beta + 1)$. And let $\overline{y}_1$ and $\overline{y}_2$ be the smallest y value satisfying the following inequalities respectively.

$$P_1(y - x_{01}|x_{01}) \geq \frac{b-m}{b+h} \tag{3.12}$$

$$P_2(y - x_{01} - 1|x_{01} + 1) \geq \frac{b-m}{b+h} \tag{3.13}$$

Note that the optimal order quantity belonging to j=1,2 are;

$$y_j^* = \begin{cases} \overline{y}_j & \text{if } \overline{y}_j \leq C(t_1); \\ C(t_1) & \text{otherwise.} \end{cases}$$

If $P_2(x) \leq P_1(x)\ \forall x$, then $P_2(\overline{y}_1 - x_{01} - 1) \leq P_1(\overline{y}_1 - x_{01} - 1)$ and $P_1(\overline{y}_1 - x_{01} - 1) < (b-m)/(b+h)$, since $\overline{y}_1$ is the smallest y value satisfying (3.16). Therefore, $\overline{y}_2 > \overline{y}_1$ and $y_2^* \geq y_1^*$, which is the desired result.
The only thing that needs to be shown is $P_2(x) \leq P_1(x)\ \forall x$, which indicates that a Negative Binomial random variable with parameters (r_2, p) dominates another Negative Binomial random variable with parameters (r_1, p) in the sense of first order stochastic dominance, where $r_2 > r_1$. This result follows from Lemma 1 in [15]. $\square$

Note that this property also holds for the optimal order quantity at any t_i; the optimal order quantity is nondecreasing as demand observed until t_i increases.

3.4 Model 2

The starting point of this model is the existence of a t_2 value minimizing the expected cost of ordering at t_2 calculated at t_1. Notation, main assumptions and *Observation* 0 of Model 1 are also valid for this model. Hence, t_1 and t_2, over which optimization is carried out, belong to the set of time points on which capacity change occurs.

Similar with Model 1, there exists a fixed order point t_1 that is known at the beginning of the period and demand information accumulates progressively until this point. Note that the manufacturer is allowed to order for the first time at t_1. At this point the manufacturer has to decide whether to place the order instantly or delay it. Our observations reveal that $E[EC^*_{t_2,x_{01}+X_{12}}]$ is discrete convex with respect to t_2, so there exists a t_2^* that minimizes the expected cost of ordering at t_2. So, the order time is set at t_2^*.

The model can be summarized as follows:

Find t_2^* that minimizes $E[EC^*_{t_2,x_{01}+X_{12}}]$.

$$t_2^* = \min_{t_1 \leq t_2 \leq 1} E[EC^*_{t_2,x_{01}+X_{12}}]$$

Note that t_2^* can be found by simple search method; by evaluating $E[EC^*_{t_2,x_{01}+X_{12}}]$ depicted in Equation (3.11) for t_2 values between t_1 and 1 with grid size 1/c due to uniformly decreasing capacity structure.
If t_2^*=t_1, order at t_1.
Otherwise, delay the order time to t_2^*.

3.5 Model 3

The first model is a static model that chooses the best order time from given two possible order times. The second model is also a static model, which chooses the order time at predetermined first possible order time based on observations so far. Contrary to the first two models, Model 3 is a dynamic model focusing on determining the best order time by considering all possible time points. Note that *Observation* 0, which highlights that the possible order points should be chosen from the time points on which capacity change occurs, is also valid for this model. Thus, c order time points are evaluated between 0 and $(c-1)/c$ that are multiples of $1/c$, where c is the total supplier capacity (See Figure 3.4).

The decision process at each time point starts with the revision of demand distribution based on realized demand thus far. It is followed by the calculation of order quantities and expected costs of ordering at that epoch and the next decision epoch. The decision process continues until ordering instantly produces an expected cost lower than the expected cost of ordering at the next decision epoch. Each time the order time is postponed, forecast accuracy is increased at the expense of losing one unit of available capacity.

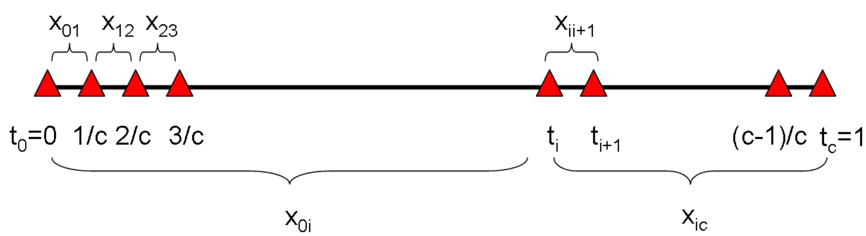

Figure 3.4: Time Line of Ordering Period for Model 3

The dynamic model starts with the calculation and comparison of the expected cost of ordering at time 0 and delaying the order until (1/c) and proceeds forwards in time until stopping condition is satisfied.

Given that it is ordered at t_i and demand of x_{0i} is observed so far, the optimal order quantity and corresponding expected cost at any t_i can be found as follows.

Select the smallest y_i value ($\overline{y}_i$) that satisfies;

$$P(y_i - x_{0i}|x_{0i}) \geq \frac{b-m}{b+h}. \tag{3.14}$$

Then, the optimal order quantity is;

$$y_i^* = \begin{cases} \overline{y}_i & \text{if } \overline{y}_i \leq C(t_i), \\ C(t_i) & \text{otherwise.} \end{cases} \tag{3.15}$$

The optimal expected total cost of ordering at t_i is;

$$EC^*_{t_i,x_{0i}} = \begin{cases} (m-b)y_i^* + b(x_{0i} + \mu_{x_{ic}|x_{0i}}) & \text{if } x_{0i} \leq C(t_i); \\ +(b+h)\sum_{x_{ic}=0}^{y_i^*-x_{0i}-1}(y_i^* - x_{0i} - x_{ic})p(x_{ic}|x_{0i}) & \\ & \\ (m-b)C(t_i) + b(x_{0i} + \mu_{x_{ic}|x_{0i}}) & \text{if } x_{0i} > C(t_i). \end{cases} \tag{3.16}$$

Given that it is delayed until t_{i+1} at t_i, the expected cost of delaying until t_{i+1} can be found as follows.

Find $\overline{y}_{i+1}$, which is the smallest value that satisfies:

$$P(y_{i+1} - x_{0,i+1}|x_{0,i+1}) \geq \frac{b-m}{b+h}. \tag{3.17}$$

The optimal order quantity at t_{i+1} under capacity restriction is;

$$y^*_{i+1} = \begin{cases} \overline{y}_{i+1} & \text{if } \overline{y}_{i+1} \leq C(t_{i+1}), \\ C(t_{i+1}) & \text{otherwise.} \end{cases} \tag{3.18}$$

$\overline{x}_{i,i+1}$ is found by selecting the largest $x_{i,i+1}$ satisfying the following inequality;

$$P(C(t_{i+1}) - x_{0i} - x_{i,i+1}|x_{0i} + x_{i,i+1}) \geq \frac{b-m}{b+h}. \tag{3.19}$$

At t_i, the optimal expected total cost of ordering at t_{i+1} is;

$$
\begin{aligned}
& E[EC^*_{t_{i+1},x_{0i}+X_{i,i+1}}] = \sum_{x_{i,i+1}=0}^{\infty} p(x_{i,i+1}|x_{0i}) EC^*_{t_{i+1},x_{0i}+x_{i,i+1}} \\
= & \sum_{x_{i,i+1}=0}^{\overline{x}_{i,i+1}} [(m-b)y^*_{i+1} + b(x_{0,i+1} + \mu_{x_{i+1,c}|x_{0,i+1}}) \\
+ & (b+h) \sum_{x_{i+1,c}=0}^{y^*_{i+1}-x_{0,i+1}-1} (y^*_{i+1} - x_{0,i+1} - x_{i+1,c}) p(x_{i+1,c}|x_{0,i+1})] p(x_{i,i+1}|x_{0i}) \\
+ & \sum_{x_{i,i+1}=\overline{x}_{i,i+1}+1}^{C(t_{i+1})-x_{0i}} [(m-b)C(t_{i+1}) + b(x_{0,i+1} + \mu_{x_{i+1,c}|x_{0,i+1}}) \\
+ & (b+h) \sum_{x_{i+1,c}=0}^{C(t_{i+1})-x_{0,i+1}-1} (C(t_{i+1}) - x_{0,i+1} - x_{i+1,c}) p(x_{i+1,c}|x_{0,i+1})] p(x_{i,i+1}|x_{0i}) \\
+ & \sum_{x_{i,i+1}=C(t_{i+1})-x_{0i}+1}^{\infty} [(m-b)C(t_{i+1}) + b(x_{0,i+1} + \mu_{x_{i+1,c}|x_{0,i+1}})] p(x_{i,i+1}|x_{0i})
\end{aligned} \tag{3.20}
$$

The dynamic process is summarized as follows:

Step 0: Begin with $i = 0$. Calculate y^*_0, $EC^*_{t_0,x_{00}}$ with Equations (3.15) and (3.16) and $E[EC^*_{t_1,x_{00}+X_{01}}]$ with Equation (3.20), where $x_{00} = 0$.
If $EC^*_{t_0,x_{00}} < E[EC^*_{t_1,x_{00}+X_{01}}]$, stop. Set time 0 as the order time.
Otherwise, continue with Step 1 by $i = 1$.

Step 1: Calculate y^*_i, $EC^*_{t_i,x_{0i}}$ with Equations (3.15) and (3.16) and
$E[EC^*_{t_{i+1},x_{0i}+X_{i,i+1}}]$ with Equation (3.20).

Step 2: If $EC^*_{t_i,x_{0i}} < E[EC^*_{t_{i+1},x_{0i}+X_{i,i+1}}]$, stop. Set t_i as the order time.
Otherwise, go back to Step 1 with $i = i + 1$.

Chapter 4

Characteristics of the Individual Models

In this chapter, we provide the characteristics of the three models that are inferred from extensive computational analysis performed and related examples. In the examples, the parameter set m=2, b=10, h=1, c=40, t_1=0.25, t_2=0.5, x_{01}=4, α=10, $\beta = 0.5$ is used unless otherwise stated.

4.1 Model 1

In this section, we present the computational analysis performed for Model 1 in order to investigate the impact of the parameters on the expected cost and the behavior of the optimal order time. Under the light of this study some interesting properties are conjectured that have not been proven analytically, yet.

Firstly, we plot an example for Property 1 presented in the previous chapter (See Figure 4.1). In this example, remaining capacity of 30 is an upper limit for the optimal order quantity at t_1. As x_{01} increases, higher demand is expected in the remaining part of the ordering period and this is reflected by an increase in the optimal order quantity until it reaches capacity limit at x_{01}=9.

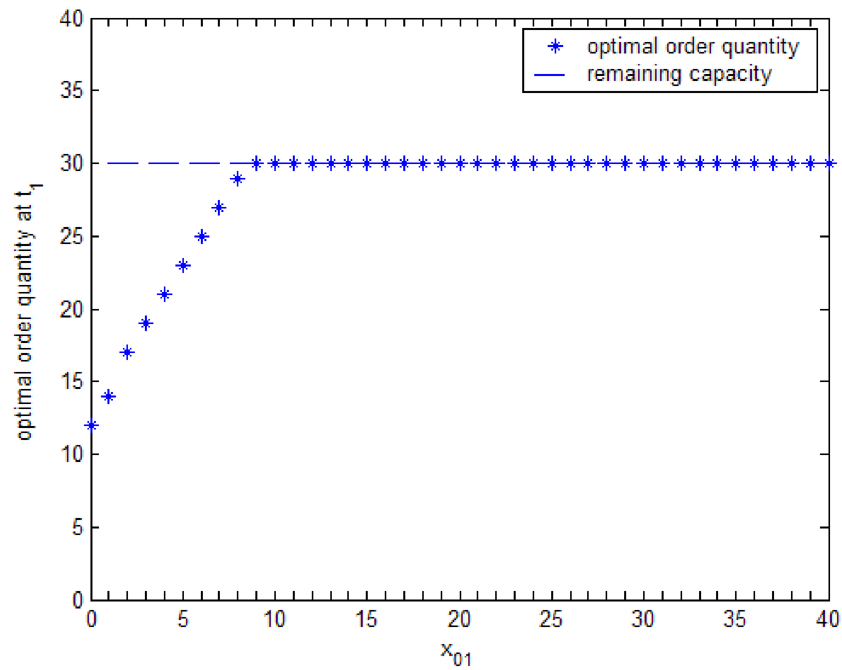

Figure 4.1: The impact of x_{01} on optimal order quantity at t_1, with m=2, b=10, h=1, c=40, t_1=0.25, α=10, $\beta = 0.5$

Observation 1: As x_{01} increases the decision maker's tendency to place the order at t_1 increases. Therefore, there exists a threshold x_{01} value above which ordering at t_1 is better than ordering at t_2.

Figure 4.2 shows the expected costs of ordering at t_1 and t_2 for our standard parameter set, when x_{01} takes values between 0 and 10. We first note that both of the expected costs are increasing with x_{01}. Since higher values of x_{01} indicates higher demand in the future, expected shortage costs increase for both of the cases. However, expected cost of ordering at t_2 shows a rather sharp increase due to less available capacity at this point. For this particular example, the threshold x_{01} value is 5. This means that delaying order time until 0.5 is better for all x_{01} values less than 5 and ordering at 0.25 is always optimal when x_{01} is 5 or higher than 5.

Observation 2: As unit shortage cost increases, the decision maker's tendency to

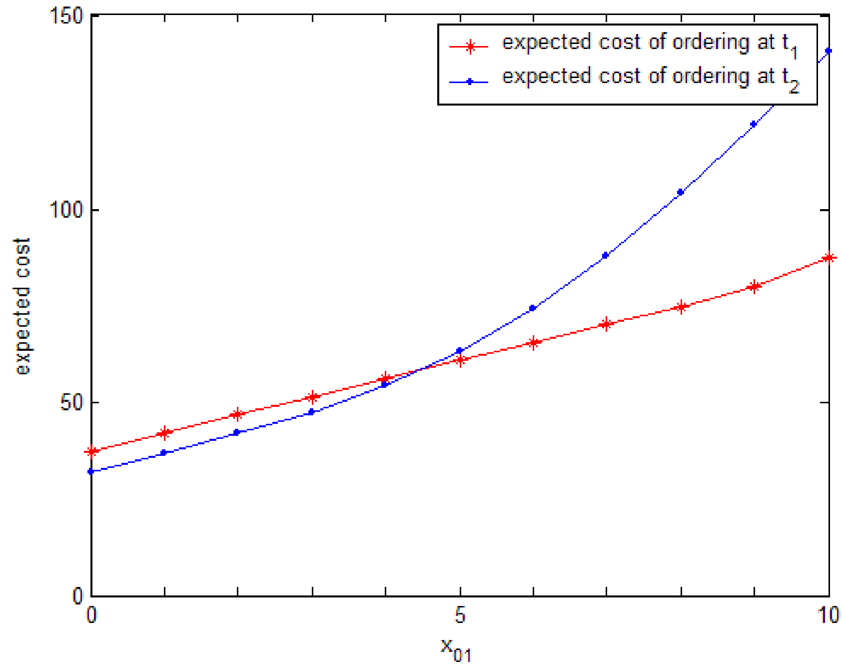

Figure 4.2: The impact of x_{01} on expected costs, with m=2, b=10, h=1, c=40, t_1=0.25, t_2=0.5, α=10, $\beta = 0.5$

place the order at t_1 increases. Thus, there exists a threshold b value above which ordering at t_1 is better than ordering at t_2.

Figure 4.3 illustrates the relationship between the expected costs of ordering at t_1 and t_2 and b, when b takes values between 5 and 11. Clearly, expected costs increase with an increase in the unit shortage cost. As b increases capacity becomes more restrictive and with this joint effect ordering at t_1 becomes less costly. For this case, the threshold b value is 14. When b is greater than equal to 14, delaying order time to 0.5 is not optimal.

Observation 3: As capacity increases, both $EC^*_{t_1,x_{01}}$ and $E[EC^*_{t_2,x_{01}+X_{12}}]$ decreases or stays the same. However, the impact on $E[EC^*_{t_2,x_{01}+X_{12}}]$ is more pronounced. Therefore, retailer's tendency to delay ordering to t_2 increases. There exists a threshold c value, above which ordering at t_2 is better than ordering at t_1.

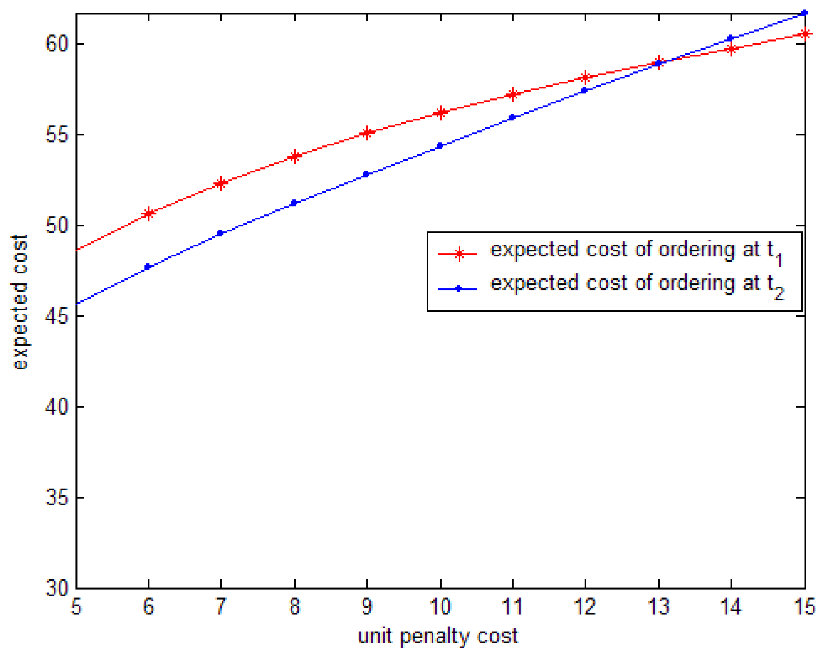

Figure 4.3: Impact of shortage cost on expected costs, with m=2, h=1, c=40, t_1=0.25, t_2=0.5, x_{01}=4, α=10, $\beta = 0.5$

Figure 4.4 shows the expected costs of ordering at t_1 and t_2 for fifteen different values of c ($c = 10, 20, \cdots, 150$). We observe that expected costs decrease sharply for the first part of the capacity increase, but the rate of decrease slows and the expected costs become constant afterwards. Since the first few units of capacity are used to materialize the best order quantity at each order point, the decrease in the expected costs are considerable. Subsequent increases in the capacity makes the expected cost of ordering at t_2 lower than expected cost of ordering at t_1, since t_2 has both sufficient capacity and more accurate demand information. For this example, ordering at t_2 is always preferred when c is greater than equal to 38.

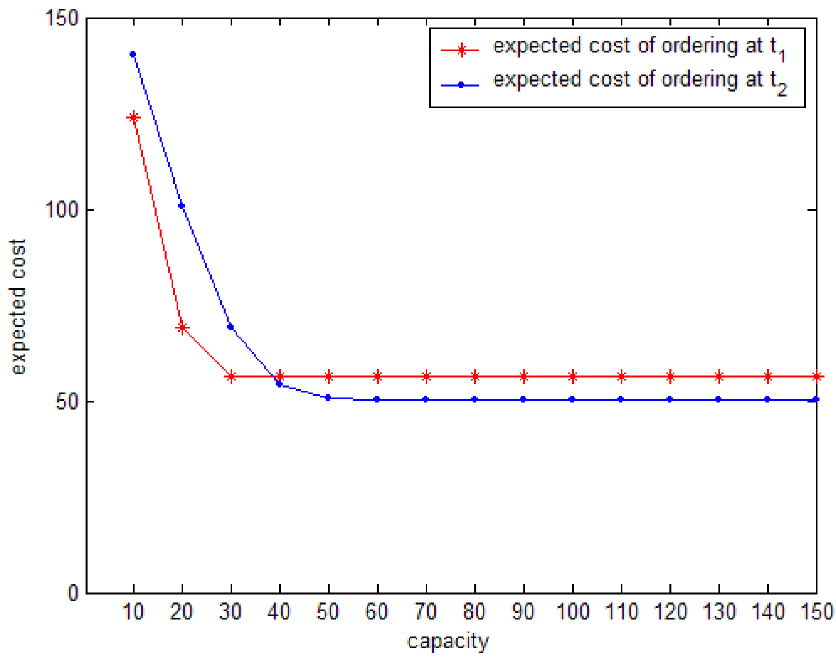

Figure 4.4: The impact of capacity on expected costs, with m=2, b=10, h=1, t_1=0.25, t_2=0.5, x_{01}=4, α=10, $\beta = 0.5$

4.2 Model 2

We now present our numerical findings for Model 2 with related examples and plots.

Observation 4: For given t_1 and x_{01} values, $E[EC^*_{t_2,x_{01}+X_{12}}]$ shows discrete convex structure as t_2 is increasing from t_1 to 1. Therefore, t_2 value that minimizes $E[EC^*_{t_2,x_{01}+X_{12}}]$ can be found by a simple search procedure. Note that t_2^* may not be unique due to discrete cost function.

This finding is similar with results of simulation study conducted at Sport Obermeyer depicted in Figure 7 in Fisher et al. [8]. Fisher et al. highlight that expected markdown and stockout costs decrease until some portion of the orders are observed and quality of information is improved. The expected cost starts to increase after it reaches the minimum in the middle part of the curve, since hereafter there is enough accumulated information but the remaining capacity is inadequate. Similar reasoning is valid for our observation. In Figure 4.5 the

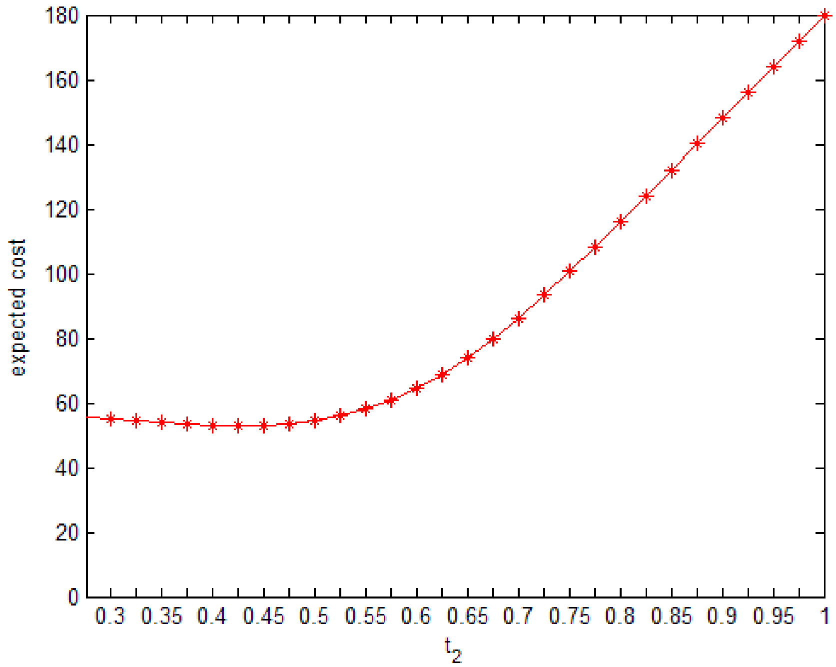

Figure 4.5: The impact of t_2 on expected cost of ordering at t_2, with m=2, b=10, h=1, c=40, t_1=0.25, x_{01}=4, α=10, $\beta = 0.5$

expected cost of ordering at t_2 is plotted, when t_2 takes values between 0.25 and 1 with increments of 0.025. We see that the expected cost is minimized when t_2 is 0.425.

Observation 5: As x_{01} increases, t_2^* approaches to t_1. Therefore, there exists a threshold x_{01} value above which $t_2^* = t_1$ and so expected cost of ordering at t_2^* is equal to expected cost of ordering at t_1.

Figure 4.6 shows the expected cost of ordering at t_1 and determined t_2^* and Figure 4.7 shows t_2^* for x_{01} values between 0 and 10. We observe that as x_{01} increases the difference between expected costs decreases progressively and becomes zero afterwards. Also, we notice that t_2^* approaches to t_1 as x_{01} increases. The intuition behind this is as follows: The increase in the observed demand increases the expectations on future demand and so delaying the order time to a further time

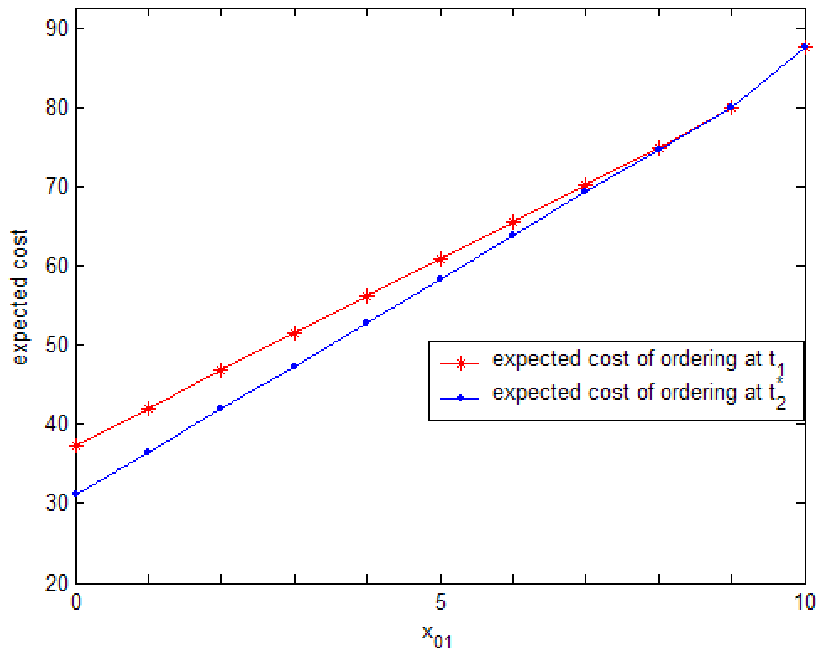

Figure 4.6: The impact of x_{01} on expected costs, with m=2, b=10, h=1, c=40, t_1=0.25, t_2=0.5, α=10, $\beta = 0.5$

point increases the expected shortage costs. For this particular case, $t_2^* = t_1$ if x_{01} is greater than equal to 9.

4.3 Model 3

In this section, we present the numerical analysis performed for Model 3 in order to find out how the model operates under different settings.

Observation 6: *Observation* 1 of Model 1 is also applicable at each decision point of this dynamic model. In other words, there exists a threshold observed demand value (x_{0i}) above which ordering instantly is better than delaying order time (1/c) units more.

Model 3 decides whether to order or continue collecting demand information

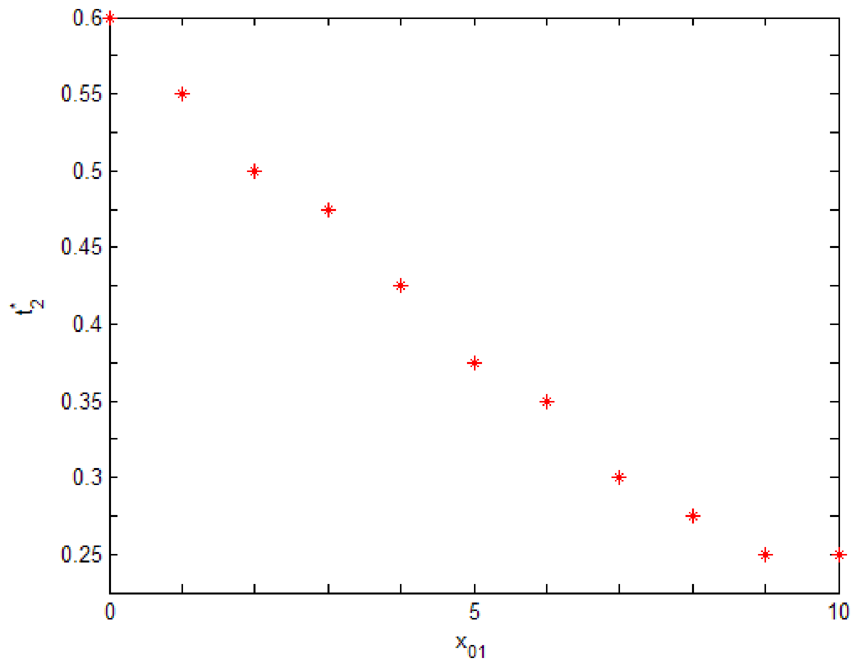

Figure 4.7: The impact of x_{01} on t_2^*, with m=2, b=10, h=1, c=40, t_1=0.25, t_2=0.5, α=10, $\beta = 0.5$

depending on this threshold value. Nothing can be said whether this threshold is always increasing or decreasing as time progresses. Figure 4.8 is plotted to show how this decision process works on an example. The plot includes threshold x_{0i} value at each time point, which are derived by comparing expected cost of ordering immediately and delaying one more time unit over x_{0i} values. For this example; if the decision maker observes 6 or more units of demand until time 0.025, he should give order immediately, otherwise he should continue observing demand. Note that under these parameters, it is not optimal to order at time 0 since the threshold value is 5. The decision maker should collect information at least until 0.025. The order decision at time 0 is given if and only if the threshold value is equal to zero and generally this is the case when shortage cost is very high or capacity is very limited. Furthermore, for this example the threshold values are firstly increasing and after remaining constant for a while they are decreasing quickly due to scarce capacity. The increase in the first part can be explained as follows: Observing high demand in a short time indicates that higher demand will be observed in the remaining part of the period due to demand learning. Since

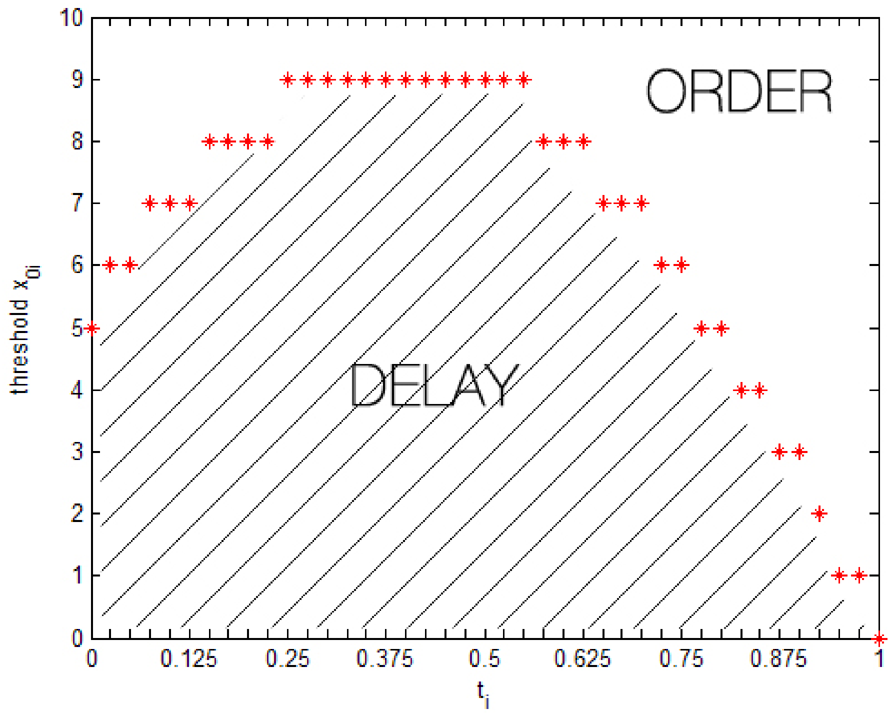

Figure 4.8: Threshold observed demand values, with m=2, b=10, h=1, c=40, α=10, $\beta = 0.5$

capacity decreases as time passes, order should be given at these early values in order to decrease shortages at the end of the period. Furthermore, the threshold demand values are decreasing in the latter parts of the period, since the possibility of realizing optimal order quantity is decreasing due to very limited capacity. The decrease through the end of the period is always true, since the capacity is very limited in the latter parts of the period.

As a next step, we focus on the impact of total available capacity and unit shortage cost on this curve. Figure 4.9 and 4.10 include threshold observed demand values for three different levels of total capacity and unit shortage cost respectively. Note that an increase (or a decrease) in one of these two parameters has reverse effect on the shape of the curve. The reason behind this is as follows: As capacity increases, the competition between capacity and accurate demand information softens and possibility of an additional shortage in the future decreases. So, the retailer can continue collecting more information by observing

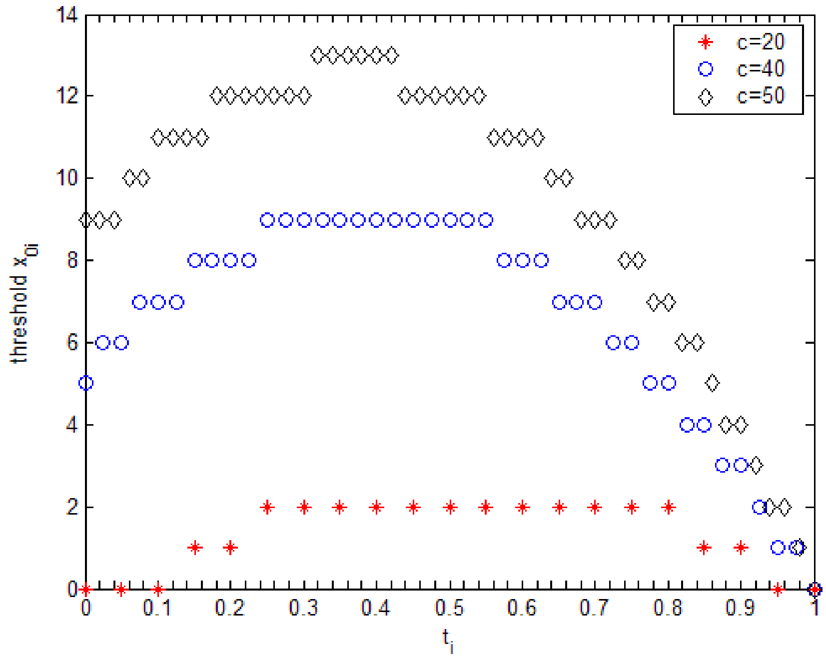

Figure 4.9: The impact of capacity on threshold observed demand values, with m=2, b=10, h=1, α=10, $\beta = 0.5$

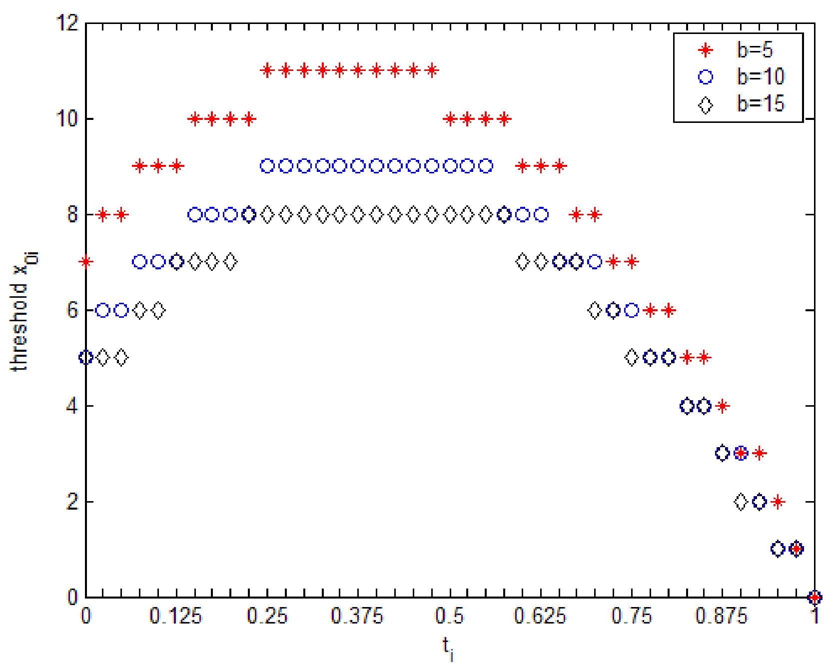

Figure 4.10: The impact of unit shortage cost on threshold observed demand values, with m=2, h=1, c=40, α=10, $\beta = 0.5$

demand at each time point as capacity increases. On the contrary as shortage cost increases, the competition between capacity and accurate demand information toughens and significance of capacity increases. Because of this, the decision maker decides to place the order at lower values of observed demand.

We analyze the impact of the initial estimate of the mean demand on the shape of the curve by Figure 4.11 using three different (α, β) pairs $((\alpha = 10,\ \beta = 0.5)$, $(\alpha = 15,\ \beta = 0.75)$, $(\alpha = 40,\ \beta = 2))$. For all pairs (α/β) kept constant at 20, while the variance decreases as α or β increases. We know that as the initial variance gets smaller, the decision maker gets more confident about his estimate of total demand and the effect of demand learning decreases. For this particular

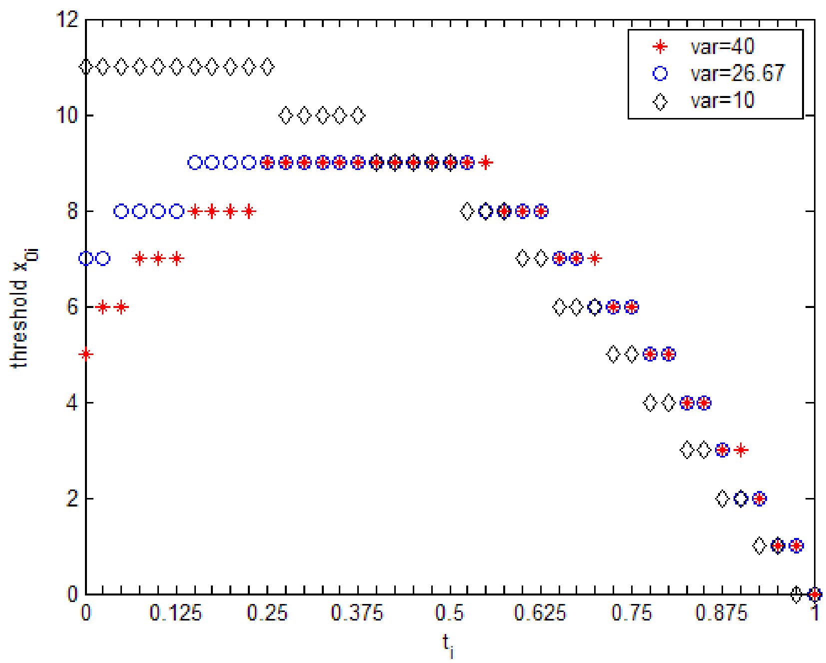

Figure 4.11: The impact of initial mean demand estimate on threshold observed demand values, with m=2, b=10, h=1, c=40

example, this is reflected by an increase in the threshold values for the initial part of the period before capacity becomes a significant constraint and by a decrease latterly. However, we cannot claim that the change in the threshold values will be always the same as in this example, when the variance decreases.

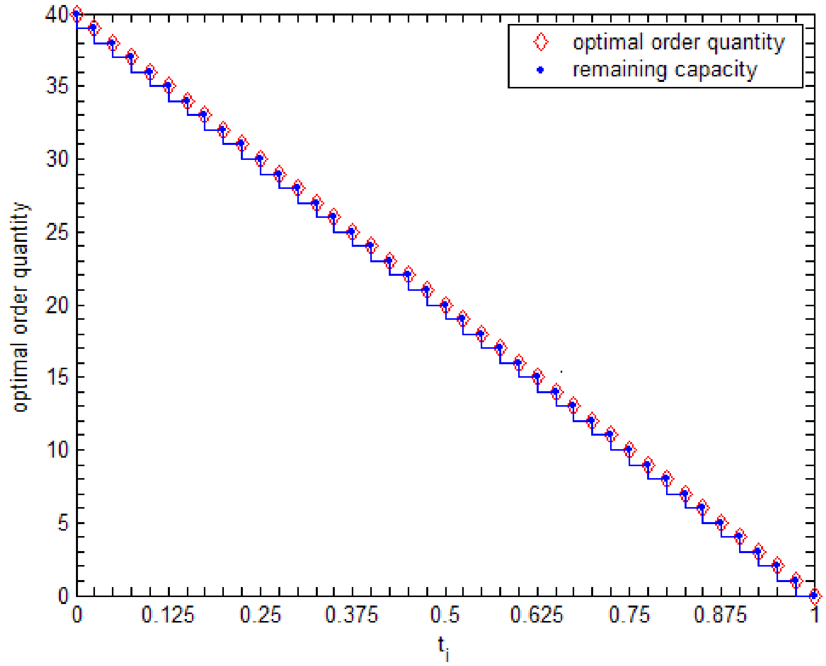

Figure 4.12: The optimal order quantities based on threshold observed demand values, with m=2, b=10, h=1, c=40, α=10, β=0.5

Finally, we investigate that the optimal order quantities at each t_i found based on the threshold observed demand value is equal to the remaining capacity at t_i. We understand that the ordering decision is delayed until the observed demand value restricts the optimal order quantity with capacity. Note that under the light of this observation, we can determine the order time by checking whether the optimal order quantity (found based on observed demand so far) hits the remaining capacity or not at each time point instead of comparing expected costs over x_{0i} values. Figure 4.12 presents an example, which clearly shows that the optimal order quantities at each time point is same as the remaining capacity at that point.

Chapter 5

Numerical Comparison of Different Models

In this chapter, we provide a summary of the results obtained from our computational studies and draw conclusions and insights from them accordingly. The aim of these studies is to investigate the performance of different models with varying levels of factors and discover how well demand learning performs under different conditions.

Thus far, we determine the best order time by assuming that the decision maker is totally unaware of the true value of the demand rate. We derive the threshold demand value, above which ordering instantly is more economical, and the optimal order quantity based on the updated Negative Binomial distribution. Note that the optimality of the decisions taken is dependent on the true value of the demand rate. In this section, we evaluate the models with the true value of the Poisson rate. In other words, firstly threshold values and optimal order quantities are found based on Bayesian approach using Negative Binomial distribution, then the expected cost of the models are calculated by the exact value of Λ. This expected cost is used for comparisons in this chapter. Also, note that the true demand rate is not known by the decision maker prior to the ordering period, so the evaluation of the models with the true Poisson rate cannot lead the decision maker at the beginning of the ordering period.

As mentioned before, the performance of the models developed significantly depends on the demand updating process and the updating process depends on the accuracy of the initial estimate of the mean demand(or parameters of the prior distribution). We assume that at the beginning of the ordering period the demand rate Λ has a Gamma distribution with parameters α and β. (α/β) is the expected value and (α/β^2) is the variance of Λ. Therefore, (α/β) gives us the initial estimate of the mean demand. When α or β is increased while keeping (α/β) at a constant value, variance decreases. Small variance indicates that the decision maker relies on the initial estimate and so is less willing to update the demand distribution with the observed demand. On the other hand, for a high variance case the weight of the accumulated demand information is higher in the update procedure. Under the light of this information, we divide our computational studies into two parts. In the first part, we investigate the impact of the parameters on the models, when the underlying initial estimate of the mean demand is true. In the second part, we discover the effects of over or under estimating the mean demand and the initial level of demand uncertainty on the performance of the models and the benefits of demand learning.

In our computational studies, we consider seven models: four of them are the models we developed and three of them are benchmark models introduced specifically for this study in order to obtain a baseline to compare with our models. The models under consideration are as follows:

1. Model 1 ($t_1 = 0.2, t_2 = 0.5$):

2. Model 1 ($t_1 = 0.2, t_2 = 0.7$):

The first two models belong to Model 1 class with the same t_1 but different t_2 values. Since t_2 significantly affects the delaying decision, its impact on the performance of the model will be observed clearly by taking $t_2 = 0.5$ in the middle of the period and $t_2 = 0.7$ at a later part of the period. These models are denoted by "Model-1(t_1, t_2)" in the tables.

3. Model 2 ($t_1 = 0.2$): The third model is Model 2 with the same t_1 value used for the previous models in order to obtain a comparison base between Model 1 and Model 2. It is denoted by "Model-2(t_1)" in the tables.

The threshold demand values and order quantities of these two models are found by Negative Binomial distribution and expectation is taken over Poisson distribution.

4. Model 3: The fourth model is Model 3 that incorporates demand learning at each time point. The threshold demand values of each decision epoch and the order quantities are derived based on the updated Negative Binomial distribution and evaluated by Poisson distribution. It is denoted by "Model-3" in the tables.

5. Model 3 with perfect information: This model is introduced in order to observe the ideal behavior of the dynamic model, when the true value of demand rate is known. The threshold demand values of each decision epoch are derived based on the Poisson distribution and evaluated also by Poisson distribution. It is denoted by "Model-3-Perf-Info" in the tables.

6. Newsboy Model: This model commits order quantity once at time 0 without observing any demand based on the initial Negative Binomial distribution. It is included in the computational studies in order to observe the effect of demand learning. It is denoted by "Newsboy" in the tables.

7. Newsboy Model with perfect information: Similar to the Newsboy Model, it decides on the order quantity once at time 0 without realizing any demand. The order quantity is found based on the correct value of Λ. This model is introduced in order to observe the effect of not utilizing demand learning when the true demand rate is known. It is denoted by "Newsboy-Perf-Info" in the tables.

Throughout our computational studies, we assume that capacity is a linearly decreasing function of the remaining time before the end of the ordering period. Note that other structures describing the decrease in the capacity can also be analyzed. For possible order times, we select the time points from the ones on which capacity change occurs. It is clear that for the uniformly decreasing capacity case, these points are multiples of $1/c$ and take values between 0 and $(c-1)/c$, including 0. Also, note that the t_1 and t_2 values used in the computational studies are chosen such that they are common factors of capacity values under investigation.

In comparisons, we use two different performance measures. First one is the percent improvement (PI) in expected cost compared with Newsboy Model that is defined as the following quantity for any model:

$$\text{PI} = \frac{\text{Exp. Cost of Newsboy-Exp. Cost of Model}}{\text{Exp. Cost of Newsboy}} \times 100.$$

For the second one, we use the realized percent improvement (RPI) with respect to Newsboy Model and Model 3 under perfect information, which is defined by the following equation:

$$\text{RPI} = \frac{\text{Exp. Cost of Newsboy-Exp. Cost of Model}}{\text{Exp. Cost of Newsboy-Exp. Cost of Model-3-Perf-Info}} \times 100.$$

The RPI gives us the percent improvement in the expected cost gained by the model under investigation compared to the base case Newsboy Model and relative to the percent improvement gained by utilizing Model 3 under perfect information.

5.1 Comparative Performance of the Models-Correct Prior Estimate of the Mean Demand

In this section, we assess the impact of the parameters on the performance of the models, when the underlying initial estimate of the mean demand is true. Throughout the analysis, we assume that m=2, b=10, h=1, c=40, $\alpha = 10$, $\beta = 0.5$, and $\Lambda = 20$ unless otherwise stated.

In Table 5.1, the percent improvements in the expected costs are shown when capacity is equal to the true value of the mean demand rate. It is clearly seen that when b=5, even Model-3-Perf-Info shows insignificant improvement and all of the models developed show worse performance than Newsboy. For this case, Model-3-Perf-Info chooses to order at time 0 as Newsboy. Since the critical fractile is low (0.5) and the demand has considerable variance, Newsboy commits for a lower order quantity and this is why Model-3-Perf-Info and Newsboy-Perf-Info is better than Newsboy. We know that Model-1 and Model-2 will produce higher expected costs than Newsboy, since they have to wait at least until 0.2 and lose an important portion of the capacity. In addition to this, we observe that Model-

	Models	b=5	b=15
Expected Cost	Newsboy	50.84	68.43
PI (percentage)	Model-1(0.2,0.5)	-8.39	-43.96
	Model-1(0.2,0.7)	-8.28	-43.96
	Model-2(0.2)	-8.60	-44.17
	Model-3	-1.28	0
	Model-3-Perf-Info	0.35	0
	Newsboy-Perf-Info	0.35	0

Table 5.1: The impact of shortage cost and limited capacity on PI in expected costs, (m=2, $h = 1$, c=20, α=10, β=0.5, Λ=20)

1(0.2, 0.5) delays the order to 0.5 if there is no demand realization until 0.2 and Model-2(0.2) delays the order to a determined time (0.4, 0.3 and 0.25 respectively) if 2 units of demand is observed until 0.2. These cause more decrease in the percent improvements. Moreover, we notice that Model-2(0.2,0.7) is better than Model-1(0.2, 0.5) and Model-2(0.2), because it place the order at 0.2 under almost all circumstances. For Model-3 it is not optimal to order at time 0. This means delaying decision is taken at least until 0.025 in spite of the decreasing capacity and this is the reason why it also shows worse performance than Newsboy. When b=15; Model-3, Model-3-Perf-Info, Newsboy, and Newsboy-Perf-Info show no improvement in the expected costs since all of them choose to order at time 0 due to limited capacity and high shortage cost and decide on the same order quantity with Newsboy, as we would expect. Note that Newsboy commits for a higher quantity compared compared with b=5 since the critical fractile is higher (0.73). Other models produce higher expected costs than Newsboy, since they have to wait at least until 0.2. Model-1s produce the same result since both of them set the order time as 0.2 for all observed demand values until 0.2. However, Model-2 delay the order time to 0.25 if no demand is realized until 0.2. And this is why it produces a lower percentage value. From this table, we can draw the conclusion that demand learning may mislead the decision maker under very limited capacity.

Since demand learning does not have any importance under very limited capacity, from now on we will not compare the results of the models for c=20.

Table 5.2 and Table 5.3 display the percent improvement and the realized percent improvement in the expected costs of the models with different values of shortage cost (5, 10, 15, 25) and capacity (40, 50). We observe that as capacity increases the improvement in Model 1(0.2,0.5) increases, while it is decreasing for Model-1(0.2,0.7), for all shortage cost values. The reason for this is as follows: As capacity increases, the model tends to decide for waiting until the second order time. For Model-1(0.2,0.5) there exists enough remaining capacity for the mean demand and more accurate demand information at 0.5. On the other hand for Model-1(0.2,0.7), there is more demand information but not enough capacity for responding it at 0.7, which consequently increases the expected shortage costs. This observation shows us that performance of Model 1 is quite sensitive to the location of t_2, which is consistent with our expectations. The percent improvement in Model-2(0.2) is also increasing with the increase in the capacity.

		Models	c=40	c=50
b=5	Expected Cost	Newsboy	50.84	50.84
	PI (percentage)	Model-1(0.2,0.5)	2.36	4.50
		Model-1(0.2,0.7)	-0.98	-1.35
		Model-2(0.2)	3.47	5.10
		Model-3	5.04	6.67
		Model-3-Perf-Info	6.42	7.99
		Newsboy-Perf-Info	0.35	0.35
b=10	Expected Cost	Newsboy	57.36	57.36
	PI (percentage)	Model-1(0.2,0.5)	1.16	4.17
		Model-1(0.2,0.7)	0.03	-0.41
		Model-2(0.2)	3.76	5.73
		Model-3	4.63	8.14
		Model-3-Perf-Info	8.43	10.77
		Newsboy-Perf-Info	1.15	1.15
b=15	Expected Cost	Newsboy	63.25	63.25
	PI (percentage)	Model-1(0.2,0.5)	4.54	8.07
		Model-1(0.2,0.7)	4.23	3.26
		Model-2(0.2)	7.26	9.42
		Model-3	7.61	11.81
		Model-3-Perf-Info	12.56	15.18
		Newsboy-Perf-Info	5.46	5.46
b=25	Expected Cost	Newsboy	68.40	68.40
	PI (percentage)	Model-1(0.2,0.5)	4.04	7.22
		Model-1(0.2,0.7)	5.43	3.22
		Model-2(0.2)	7.98	9.89
		Model-3	7.22	12.66
		Model-3-Perf-Info	14.19	17.17
		Newsboy-Perf-Info	7.02	7.02

Table 5.2: The impact of shortage cost and capacity on PI in expected costs, (m=2, $h = 1$, α=10, β=0.5, Λ=20)

		Models	c=40	c=50
b=5	Expected Cost	Model-3-Perf-Info	47.58	46.78
		Newsboy	50.84	50.84
	RPI (percentage)	Model-1(0.2,0.5)	36.75	56.34
		Model-1(0.2,0.7)	-15.31	-16.87
		Model-2(0.2)	54.02	63.77
		Model-3	78.59	83.46
		Newsboy-Perf-Info	5.47	4.39
b=10	Expected Cost	Model-3-Perf-Info	52.53	51.18
		Newsboy	57.36	57.36
	RPI (percentage)	Model-1(0.2,0.5)	13.73	38.70
		Model-1(0.2,0.7)	0.41	-3.79
		Model-2(0.2)	44.57	53.16
		Model-3	54.90	75.60
		Newsboy-Perf-Info	13.70	10.72
b=15	Expected Cost	Model-3-Perf-Info	55.31	53.65
		Newsboy	63.25	63.25
	RPI (percentage)	Model-1(0.2,0.5)	36.15	53.17
		Model-1(0.2,0.7)	33.65	21.47
		Model-2(0.2)	57.80	62.06
		Model-3	60.56	77.82
		Newsboy-Perf-Info	43.42	35.93
b=25	Expected Cost	Model-3-Perf-Info	58.70	56.66
		Newsboy	68.40	68.40
	RPI (percentage)	Model-1(0.2,0.5)	28.49	42.05
		Model-1(0.2,0.7)	38.28	18.78
		Model-2(0.2)	56.28	57.64
		Model-3	50.90	73.76
		Newsboy-Perf-Info	49.47	40.89

Table 5.3: The impact of shortage cost and capacity on RPI in expected costs, (m=2, $h = 1$, α=10, β=0.5, Λ=20)

		Models	c=40	c=50
b=10	RPI (percentage)	Model-1(0.2,0.5)	13.73	38.70
		Model-1(0.2,0.7)	0.41	-3.79
		Model-2(0.2)	44.57	53.16
		Model-3	54.90	75.60
		Model-3(0.2)	59.17	75.69
b=25	RPI (percentage)	Model 1(0.2,0.5)	28.49	42.05
		Model-1(0.2,0.7)	38.28	18.78
		Model-2(0.2)	56.28	57.64
		Model-3	50.90	73.76
		Model-3(0.2)	63.75	74.41

Table 5.4: The impact of t_1 on RPI in expected cost of Model-3, (m=2, $h = 1$, α=10, β=0.5, Λ=20)

As capacity increases, the model can determine order times different than 0.2 for higher observed demand values until 0.2 and thereby takes more advantage of demand learning. Similarly, increase in capacity leads to increase in benefits gained from Model-3. When capacity is higher, the model decides on delaying the order for higher threshold total observed demand values at each time point. Thus, decisions are based on more accurate demand estimate and more benefit of demand learning is gained. We observe that Model-3 outperforms other models except for one case, when $c = 40$ and $b = 25$. Since capacity is low and shortage cost is very high, the model triggers ordering decision at early times for low observed demand values. These decisions based on less demand information increase the expected cost. If we define a time point (t_1) for Model-3, until which information accumulates and ordering is not allowed before, and let the dynamic process start hereafter, then we will see that Model-3 (using same t_1 with Model-1 and Model-2) will perform better than all of the models under low capacity and high shortage cost cases too (See Table 5.4, the new model is denoted by Model-3(0.2)). Collecting information for a certain amount of time without evaluating the ordering decision prevents the decision maker to order in the initial part of the period, during which uncertainty in demand still remains, and this lowers the expected cost. Moreover, we notice that the improvement in Newsboy-Perf-Info increases with the shortage cost, since it decides to order more, expected shortage costs decrease. On the other hand, we observe that PI of this model does not change with the increase in capacity whereas RPI decreases. The reason is that

		Models	c=40	c=50
t_1	Expected Cost	Newsboy	57.36	57.36
0.2	PI (percentage)	Model-1(t_1,0.5)	1.16	4.17
		Model-1(t_1,0.7)	0.03	-0.41
		Model-2(t_1)	3.76	5.73
0.3	PI (percentage)	Model-1(t_1,0.5)	3.84	5.42
		Model-1(t_1,0.7)	3.06	0.72
		Model-2(t_1)	5.39	6.51
0.4	PI (percentage)	Model-1(t_1,0.5)	5.34	6.74
		Model-1(t_1,0.7)	4.79	5.05
		Model-2(t_1)	5.62	7.84
0.5	PI (percentage)	Model-1(t_1,0.7)	-4.10	7.04
		Model-2(t_1)	-3.76	8.52

Table 5.5: The impact of t_1 on PI in expected costs, (m=2, b=10, h = 1, α=10, β=0.5, Λ=20)

the order quantities of both Newsboy and Newsboy-Perf-Info stay the same with the change in capacity, but the expected cost of Model-3-Perf-Info is lower for higher capacity.

To summarize, this analysis leads us to draw the conclusion that the value of demand learning and improvements gained from the models increase, as capacity increases.

Table 5.5 presents the percent improvements in the expected costs of the models including a first demand accumulation point (t_1), when t_1 takes on values 0.2, 0.3, 0.4, and 0.5 and capacity takes on values 40 and 50. As t_1 is shifted through the end of the period, the uncertainty of the demand at t_1 reduces. Note that Model-1 and Model-2 decides on the order time at t_1. Then, the order time will be determined on more accurate demand information for a higher t_1 value and this lowers the expected costs. However, after a time point the expected cost starts to increase since hereafter there is sufficient demand information but inadequate capacity. This is why the RPI in the expected costs of Model-1s and Model-2 increase between t_1=0.2 and t_1=0.4 and become negative at t_1=0.5, when c=40. Moreover, we observe that Model-2 produces higher RPIs than Model-1s for all of the cases. This is reasonable, since it has the flexibility to choose the order

		Models	c=40	c=50
α=10, β=0.5 variance=40	Expected Cost	Model-3-Perf-Info	52.53	51.18
		Newsboy	57.36	57.36
	RPI (percentage)	Model-2(0.2)	44.57	53.16
		Model-3	54.90	75.60
		Newsboy-Perf-Info	13.70	10.72
α=40, β=2 variance=10	Expected Cost	Model-3-Perf-Info	52.53	51.18
		Newsboy	56.70	56.70
	RPI (percentage)	Model-2(0.2)	76.06	78.53
		Model-3	92.18	95.98
		Newsboy-Perf-Info	0	0

Table 5.6: The impact of initial mean demand estimate on RPI in expected costs, (m=2, b=10, h = 1, Λ=20)

time by considering more possible order times based on the observations until t_1.

Since the performance of Model-1 strongly depends on chosen t_1 and t_2 values, from now on we will not include it in our computational analysis.

Table 5.6 studies the impact of the variance of the initial mean demand estimate on the expected costs of the models. While the initial estimate of the mean demand is fixed at 20, the variance of the estimate takes on values 40 and 10. We know that in Bayesian approach, high variance of the initial mean demand estimate indicates that the decision maker is quite unsure about his estimate and gives more weight to the observed demand in the update procedure and vice versa for low variance. The findings in Table 5.5 are consistent with this information. When the variance is 40, demand learning may mislead the decision maker since the demand estimate will quickly respond to the observed demand. However, the update procedure is relatively robust to the observed demand when the variance is 10. Hence, the models with demand learning take decisions in accordance with the true mean demand estimate and produce lower expected costs and higher RPIs in the expected costs. As seen from the table, all of the models developed are better off when the variance is lower. Besides, when the variance is lower, Newsboy decides on the same order quantity with Newsboy-Perf-Info and this leads Newsboy Perf Info to produce zero RPI.

Notice that the maximum improvements in the expected costs are achieved for Model-3, since it uses demand learning continuously for determining the best order time and the corresponding best order quantity. The studies in this section illustrate that the benefits of demand learning under the true initial point estimate are higher; when the available capacity is high and the initial level of demand uncertainty is low.

5.2 Comparative Performance of the Models-Incorrect Prior Estimate of the Mean Demand

The objective in this section is to investigate the effects of the initial estimate of the mean demand on demand learning and the expected cost of implementing the models. Throughout the analysis, we keep the initial estimate of the mean demand constant and vary the true value of the demand rate and the variance of the initial mean demand estimate. Note that the models are evaluated with the true value of the demand rate.

We analyze the impact of the initial level of uncertainty and true demand rate in detail on the performance of demand learning. In the analysis, we also include the effect of capacity by taking $c = 40$ in Table 5.7 and Table 5.8 and $c = 50$ in Table 5.9 and Table 5.10. The (α, β) pair takes values (5, 0.25), (10, 0.5), (15, 0.75), (25, 1.25), (40, 2) for which the initial estimate of the mean demand is constant at 20. But the variance takes values 80, 40, 26.67, 16, and 10, respectively. Firstly, we analyze the change in the expected costs of Newsboy and Newsboy-Perf-Info as variance increases, which is as follows: For $\Lambda = 20$ case, as the variance of the initial mean demand estimate decreases, the variance approaches to the variance of the true demand. Hence, the optimal order quantity and consequently expected cost of Newsboy firstly decrease and then become equal with Newsboy-Perf-Info. As mentioned, the optimal order quantity found for Newsboy decreases as the variance decreases. For $\Lambda = 10$, this is reflected as a decrease in the expected cost of Newsboy, since the difference between order amount and the true demand rate becomes smaller. Consequently, the percent improvement of Newsboy-Perf-Info decreases. On the other hand for $\Lambda = 30$, as the variance decreases, the expected cost of Newsboy increases due to increase in expected shortage cost since the difference between order amount and the true demand rate increases and consequently the percent improvement of Newsboy-Perf-Info increases.

Note that under both of the capacity values the results of $\Lambda = 20$ are similar to the results in Table 5.5; improvement gained decreases with the increase in the variance of the initial mean demand estimate. The reason is that when the variance is high the update procedure quickly responds to the observed demand values and this may lead the decision maker to take wrong decisions. This is in contrast for the cases when Λ is 10 and 30. Since, higher variance allows the updating procedure to identify the inaccuracy of the estimate quickly and update it for the subsequent decision epoch. We observe that the PI and RPI is always higher for $\Lambda = 10$ and $\Lambda = 30$ compared to the PI and RPI for $\Lambda = 20$, when the variance is high. Thus, we conclude that learning is most beneficial when the initial estimate of the mean demand is inaccurate and the variance of the estimate is high. Again, Model 3 produces higher percentage improvement values for all of the cases, since it incorporates demand learning continuously and update the mean demand estimate at each decision epoch. Additionally, we notice that the learning effect is more pronounced when $c = 50$. Because delaying is a preferable decision due to more available capacity and this makes the decision maker to learn more about the demand. Also, we note that the improvements are more noticeable, when the mean demand is over-estimated. But, we do not have any intuition about this observation now.

Under the light of this analysis, we infer that the models perform better when the mean demand is overestimated and the capacity is higher.

		Models	(α, β)				
			(5,0.25)	(10,0.5)	(15,0.75)	(25,1.25)	(40,2)
Λ=10	Expected Cost	Model-3-Perf-Info	26.42	26.42	26.42	26.42	26.42
		Newsboy	65.00	62.00	62.00	59.00	59.00
	RPI (percentage)	Model-2(0.2)	80.67	74.47	71.32	63.52	59.45
		Model-3	92.95	89.84	88.38	82.51	79.88
		Newsboy-Perf-Info	85.96	84.78	84.78	83.37	83.37
Λ=20	Expected Cost	Model-3-Perf-Info	52.53	52.53	52.53	52.53	52.53
		Newsboy	58.64	57.36	57.36	56.70	56.70
	RPI (percentage)	Model-2(0.2)	28.48	44.57	60.83	69.25	76.06
		Model-3	30.71	54.90	68.54	88.43	92.18
		Newsboy-Perf-Info	31.72	13.70	13.70	0	0
Λ=30	Expected Cost	Model-3-Perf-Info	78.60	78.60	78.60	78.60	78.60
		Newsboy	105.41	111.68	111.68	118.42	118.42
	RPI (percentage)	Model-2(0.2)	57.33	58.44	52.35	54.59	50.26
		Model-3	75.33	72.49	68.17	62.58	58.62
		Newsboy-Perf-Info	93.51	94.74	94.74	95.63	95.63

Table 5.7: The impact of initial level of uncertainty and true demand rate on RPI in expected costs when initial mean demand estimate is 20, (m=2, b=10, h=1, c = 40)

		Models	(α,β)				
			(5,0.25)	(10,0.5)	(15,0.75)	(25,1.25)	(40,2)
Λ=10	Expected Cost	Newsboy	65.00	62.00	62.00	59.00	59.00
	PI (percentage)	Model-3-Perf-Info	59.35	57.38	57.38	55.22	55.22
		Model-2(0.2)	47.88	42.73	40.92	35.07	32.83
		Model-3	55.16	51.55	50.72	45.56	44.10
		Newsboy-Perf-Info	51.02	48.65	48.65	46.04	46.04
Λ=20	Expected Cost	Newsboy	58.64	57.36	57.36	56.70	56.70
	PI (percentage)	Model-3-Perf-Info	10.42	8.43	8.43	7.36	7.36
		Model-2(0.2)	2.97	3.76	5.13	5.09	5.60
		Model-3	3.20	4.63	5.78	6.51	6.78
		Newsboy-Perf-Info	3.30	1.15	1.15	0	0
Λ=30	Expected Cost	Newsboy	105.41	111.68	111.68	118.42	118.42
	PI (percentage)	Model-3-Perf-Info	25.44	29.62	29.62	33.63	33.63
		Model-2(0.2)	14.58	17.31	15.51	18.36	16.90
		Model-3	19.16	21.48	20.19	21.05	19.71
		Newsboy-Perf-Info	23.79	28.06	28.06	32.16	32.16

Table 5.8: The impact of initial level of uncertainty and true demand rate on PI in expected costs when initial mean demand estimate is 20, (m=2, b=10, h=1, $c = 40$)

		Models	(α,β)				
			(5,0.25)	(10,0.5)	(15,0.75)	(25,1.25)	(40,2)
Λ=10	Expected Cost	Model-3-Perf-Info	25.74	25.74	25.74	25.74	25.74
		Newsboy	65.00	62.00	62.00	59.00	59.00
	RPI (percentage)	Model-2(0.2)	85.06	80.39	77.59	71.22	67.96
		Model-3	95.94	94.04	91.47	89.84	86.44
		Newsboy-Perf-Info	84.46	83.18	83.18	81.66	81.66
Λ=20	Expected Cost	Model-3-Perf-Info	51.18	51.18	51.18	51.18	51.18
		Newsboy	58.64	57.36	57.36	56.70	56.70
	RPI (percentage)	Model-2(0.2)	38.21	53.16	66.42	72.93	78.53
		Model-3	63.41	75.60	84.78	90.33	95.98
		Newsboy-Perf-Info	25.99	10.72	10.72	0	0
Λ=30	Expected Cost	Model-3-Perf-Info	76.59	76.59	76.59	76.59	76.59
		Newsboy	105.41	111.68	111.68	118.42	118.42
	RPI (percentage)	Model-2(0.2)	72.21	72.54	66.41	65.75	60.31
		Model-3	81.41	83.17	80.37	78.33	72.26
		Newsboy-Perf-Info	86.99	89.32	89.32	91.04	91.04

Table 5.9: The impact of initial level of uncertainty and true demand rate on RPI in expected costs when initial mean demand estimate is 20, (m=2, b=10, h=1, $c = 50$)

		Models	(α,β)				
			(5,0.25)	(10,0.5)	(15,0.75)	(25,1.25)	(40,2)
Λ=10	Expected Cost	Newsboy	65.00	62.00	62.00	59.00	59.00
	PI (percentage)	Model-3-Perf-Info	60.40	58.49	58.49	56.38	56.38
		Model-2(0.2)	51.38	47.02	45.38	40.15	38.31
		Model-3	57.95	55.00	53.50	50.65	48.73
		Newsboy-Perf-Info	51.02	48.65	48.65	46.04	46.04
Λ=20	Expected Cost	Newsboy	58.64	57.36	57.36	56.70	56.70
	PI (percentage)	Model-3-Perf-Info	12.71	10.77	10.77	9.73	9.73
		Model-2(0.2)	4.86	5.73	7.16	7.10	7.64
		Model-3	8.06	8.14	9.13	8.79	9.34
		Newsboy-Perf-Info	3.30	1.15	1.15	0	0
Λ=30	Expected Cost	Newsboy	105.41	111.68	111.68	118.42	118.42
	PI (percentage)	Model-3-Perf-Info	27.34	31.42	31.42	35.32	35.32
		Model-2(0.2)	19.74	22.79	20.87	23.22	21.30
		Model-3	22.26	26.13	25.25	27.67	25.53
		Newsboy-Perf-Info	23.79	28.06	28.06	32.16	32.16

Table 5.10: The impact of initial level of uncertainty and true demand rate on PI in expected costs when initial mean demand estimate is 20, (m=2, b=10, h=1, $c = 50$)

Chapter 6

Conclusion

In this thesis, we study the problem of determining the order time for a seasonal products manufacturer in an environment where demand learning is possible. The manufacturer has a single opportunity for ordering and its supplier's capacity decreases as time progresses. The distribution of the demand is updated by Bayesian approach at each order opportunity in order to improve the quality of the forecasts. A standard form of Bayesian approach is used for the demand model, which assumes that demand in the whole ordering period is distributed by Poisson with an unknown parameter. The prior distribution of the unknown parameter is Gamma which produces Negative Binomial distribution for unconditional demand. Three different models are developed that incorporates demand learning, each having a different assumption in terms of existing possible order times. The first model chooses the best order time from two defined order times prior to the ordering period. The second model sets the best order time depending on the first predetermined order time and observed demand so far. Both of the models update the demand distribution at the first predetermined order time and decide on the order time based on the new distribution. The last model considers each time point as a possible order time, continuously updates the demand information and chooses the best order time accordingly. In addition, we have conducted a series of computational studies to investigate the operating properties of the models we developed and to assess the performance of the models under different parameter settings. Some key conclusions and insights drawn from these studies are as follows. Firstly, we observe that dynamic model that incorporates continuous demand learning shows better performance than the other models. Secondly, we observe that demand learning is most beneficial, when the initial estimate of the mean demand is inaccurate and has high level of uncertainty. Also, the benefits of learning is more pronounced when the capacity is high and mis-estimation of the mean demand is significant.

We also identify a number of cases for which the study can be extended. First one is using an increasing unit purchasing cost over time in the expected cost. Since the supplier has limited capacity, the increase in the purchase cost through the end of the ordering period is reasonable. It will be useful in terms of observing a more pronounced trade-off between the capacity and more accurate demand information. Second one is analyzing the models under nonlinear decreasing capacity. This will also lead to a different trade-off structure between the demand and the remaining capacity.

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Appendix A

Results of Computational Studies

In this section, we provide the exact values of the expected costs.

b	Models	c=20	c=40	c=50
5	Model-3-Perf-Info	50.66	47.58	46.78
	Newsboy	50.84	50.84	50.84
	Model-1(0.2, 0.5)	55.11	49.64	48.55
	Model-1(0.2,0.7)	55.05	51.34	51.52
	Model-2(0.2)	55.21	49.08	48.25
	Model-3	51.49	48.28	47.45
	Newsboy-Perf-Info	50.66	50.66	50.66
10	Model-3-Perf-Info	59.54	52.53	51.18
	Newsboy	59.54	57.36	57.36
	Model-1(0.2, 0.5)	76.60	56.70	54.97
	Model-1(0.2,0.7)	76.60	57.34	57.60
	Model-2(0.2)	76.65	55.21	54.08
	Model-3	59.54	54.71	52.69
	Newsboy-Perf-Info	59.54	56.70	56.70
15	Model-3-Perf-Info	68.43	55.31	53.65
	Newsboy	68.43	63.25	63.25
	Model-1(0.2, 0.5)	98.51	60.38	58.15
	Model-1(0.2,0.7)	98.51	60.58	61.19
	Model-2(0.2)	98.65	58.66	57.29
	Model-3	68.43	58.44	55.78
	Newsboy-Perf-Info	68.43	59.80	59.80
25	Model-3-Perf-Info	86.19	58.70	56.66
	Newsboy	86.19	68.40	68.40
	Model-1(0.2, 0.5)	142.58	65.64	63.47
	Model-1(0.2,0.7)	142.58	64.69	66.20
	Model-2(0.2)	142.58	62.94	61.63
	Model-3	86.19	63.46	59.74
	Newsboy-Perf-Info	86.19	63.60	63.60

Table A.1: The impact of capacity and shortage cost on expected costs, (m=2, $h = 1$, α=10, β=0.5, Λ=20)

b	Models	c=40	c=50
10	Model-2(0.2)	62.94	61.63
	Model-3	54.71	52.69
	Model-3(0.2)	54.50	52.69
25	Model-2(0.2)	55.21	54.08
	Model-3	63.46	59.74
	Model-3(0.2)	62.22	59.66

Table A.2: The impact of t_1 on Model 3, (m=2, $h = 1$, α=10, β=0.5, Λ=20)

t_1	Models	c=40	c=50
	Newsboy	57.36	57.36
0.2	Model-1(t_1, 0.5)	56.70	54.97
	Model-1(t_1,0.7)	57.34	57.60
	Model-2(t_1)	55.21	54.08
0.3	Model-1(t_1, 0.5)	55.16	54.25
	Model-1(t_1,0.7)	55.61	56.95
	Model-2(t_1)	54.27	53.63
0.4	Model-1(t_1, 0.5)	54.30	53.49
	Model-1(t_1,0.7)	54.62	54.47
	Model-2(t_1)	54.14	52.87
0.5	Model-1(t_1,0.7)	59.71	53.32
	Model-2(t_1)	59.52	52.48

Table A.3: The impact of t_1 on expected costs, (m=2, b=10, $h = 1$, α=10, β=0.5, Λ=20)

α, β	Models	c=40	c=50
	Model-3-Perf-Info	52.53	51.18
α=10, β=0.5	Newsboy	57.36	57.36
variance=40	Model-2(0.2)	55.21	54.08
	Model-3	54.71	52.69
	Newsboy-Perf-Info	56.70	56.70
	Model-3-Perf-Info	52.53	51.18
α=40, β=2	Newsboy	56.70	56.70
variance=10	Model-2(0.2)	53.53	52.37
	Model-3	52.86	51.41
	Newsboy-Perf-Info	56.70	56.70

Table A.4: The impact of initial mean demand estimate on expected costs, (m=2, b=10, $h = 1$, Λ=20)

Λ	Models	$\alpha = 5, \beta = 0.25$	$\alpha = 10, \beta = 0.5$	$\alpha = 15, \beta = 0.75$	$\alpha = 25, \beta = 1.25$	$\alpha = 40, \beta = 2$
	Model-3-Perf-Info	26.42	26.42	26.42	26.42	26.42
	Newsboy	65.00	62.00	62.00	59.00	59.00
10	Model-2(0.2)	33.88	35.50	36.63	38.31	39.63
	Model-3	29.14	30.04	30.56	32.12	32.98
	Newsboy-Perf-Info	31.84	31.84	31.84	31.84	31.84
	Model-3-Perf-Info	52.53	52.53	52.53	52.53	52.53
	Newsboy	58.64	57.36	57.36	56.70	56.70
20	Model-2(0.2)	56.90	55.21	54.42	53.81	53.53
	Model-3	56.76	54.71	54.05	53.01	52.86
	Newsboy-Perf-Info	56.70	56.70	56.70	56.70	56.70
	Model-3-Perf-Info	78.60	78.60	78.60	78.60	78.60
	Newsboy	105.41	111.68	111.68	118.42	118.42
30	Model-2(0.2)	90.04	92.35	94.36	96.68	98.40
	Model-3	85.21	87.70	89.13	93.50	95.07
	Newsboy-Perf-Info	80.34	80.34	80.34	80.34	80.34

Table A.5: The impact of initial level of uncertainty and true demand rate on expected costs when initial mean demand estimate is 20, (m=2, b=10, h=1, $c = 40$)

Λ	Models	$\alpha = 5, \beta = 0.25$	$\alpha = 10, \beta = 0.5$	$\alpha = 15, \beta = 0.75$	$\alpha = 25, \beta = 1.25$	$\alpha = 40, \beta = 2$
	Model-3-Perf-Info	25.74	25.74	25.74	25.74	25.74
	Newsboy	65.00	62.00	62.00	59.00	59.00
10	Model-2(0.2)	31.60	32.85	33.87	35.31	36.40
	Model-3	27.33	27.90	28.83	29.12	30.25
	Newsboy-Perf-Info	31.84	31.84	31.84	31.84	31.84
	Model-3-Perf-Info	51.18	51.18	51.18	51.18	51.18
	Newsboy	58.64	57.36	57.36	56.70	56.70
20	Model-2(0.2)	55.79	54.08	53.26	52.68	52.37
	Model-3	53.91	52.69	52.12	51.72	51.41
	Newsboy-Perf-Info	56.70	56.70	56.70	56.70	56.70
	Model-3-Perf-Info	76.59	76.59	76.59	76.59	76.59
	Newsboy	105.41	111.68	111.68	118.42	118.42
30	Model-2(0.2)	84.60	86.22	88.38	90.92	93.19
	Model-3	81.95	82.50	83.48	85.65	88.19
	Newsboy-Perf-Info	80.34	80.34	80.34	80.34	80.34

Table A.6: The impact of initial level of uncertainty and true demand rate on expected costs when initial mean demand estimate is 20, (m=2, b=10, h=1, $c = 50$)